Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 43 (1997)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: BOTT-CHERN FORMS AND ARITHMETIC INTERSECTIONS

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Kurzfassung

DOI: https://doi.org/10.5169/seals-63270

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L'Enseignement Mathématique, t. 43 (1997), p. 33–54

BOTT-CHERN FORMS AND ARITHMETIC INTERSECTIONS

by Harry TAMVAKIS

ABSTRACT. Let $\overline{\mathcal{E}} : 0 \to \bar{S} \to \bar{E} \to \bar{Q} \to 0$ be a short exact sequence of hermitian vector bundles with metrics on S and Q induced from that on E. We compute the Bott-Chern form $\widetilde{\phi}(\overline{\mathcal{E}})$ corresponding to any characteristic class ϕ, assuming $\bar{E}$ is projectively flat. The result is used to obtain a new presentation of the Arakelov Chow ring of the arithmetic Grassmannian.

1. INTRODUCTION

Arakelov theory is an intersection theory for varieties over rings $\mathcal{O}_F$ of algebraic integers, analogous to the usual one over fields. The fundamental idea is that in order to have a good theory of intersection numbers, one has to include information at the infinite primes.

The work of Arakelov in dimension two has been generalized by Gillet and Soulé to higher dimensional *arithmetic varieties* X, by which we mean regular, projective and flat schemes over $\operatorname{Spec} \mathbf{Z}$. They define an *arithmetic Chow ring* $\widehat{CH}(X)_{\mathbf{Q}}$ whose elements are represented by cycles on X together with Green currents on $X(\mathbf{C})$. The theory is a blend of arithmetic, algebraic geometry and complex hermitian geometry. For example, the Faltings height of an arithmetic variety X is realized as an "arithmetic degree" with respect to a hermitian line bundle over X.

A *hermitian vector bundle* $\bar{E} = (E, h)$ over X is an algebraic vector bundle E on X together with a hermitian metric h on the corresponding holomorphic vector bundle $E(\mathbf{C})$ on the complex manifold $X(\mathbf{C})$. To such an object one associates arithmetic Chern classes $\widehat{c}(\bar{E})$ with values in $\widehat{CH}(X)$. These satisfy most of the usual properties of Chern classes, with one exception: given a short exact sequence of hermitian vector bundles