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Artikel: AN ASYMPTOTIC FREIHEITSSATZ FOR FINITELY GENERATED GROUPS

Autor: Cherix, Pierre-Alain / SCHAEFFER, Gilles

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With that definition, the precise statement of Champetier's theorem is:

THEOREM (Champetier). *Let C be a positive constant. For every $\epsilon > 0$, there exists an integer n_0 such that for every presentation $\Gamma = \langle X \mid R \rangle$, with $\#R = m$, satisfying $A(C)$ and $n_0 \leq \inf\{|r| \mid r \in R\}$, the following inequalities hold:*

$$\frac{\sqrt{2(\#X) - 1}}{\#X} \leq \|h_S\| \leq \left(1 + \frac{\epsilon}{2}\right) \frac{\sqrt{2(\#X) - 1}}{\#X}.$$

Assume the presentation satisfies a θ-condition: then $I(\Delta) < \theta\#(\Delta) = \theta(I(\Delta) + |\omega|)$, for any reduced diagram associated with ω. As

$$\sum_{i=1}^{m} l_i |r_i| = \sum_{2\text{-cell } f \subset \Delta} \#(f) \leq 2I(\Delta) + E(\Delta),$$

it is easy to see that the θ-condition implies $A(\frac{2\theta}{1-\theta} + 1)$. So Champetier's theorem and the genericity of the θ-condition imply:

COROLLARY 5.3. *For every $\epsilon > 0$, every fixed $\#X = k$ and every fixed $\#R = m$, $\|h_S\|$ is generically close to $\frac{\sqrt{2(\#X)-1}}{\#X}$.*

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Pierre-Alain Cherix

School of mathematics
University of New South Wales
Sydney, 2052
Australia
e-mail: pacherix@maths.unsw.edu.au

Gilles Schaeffer

LaBRI
Université Bordeaux I
351 Cours de la Libération
33405 Talence Cedex
France
e-mail: Gilles.Schaeffer@labri.u-bordeaux.fr