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the original system (2) which establishes once again Theorem A.2, (ii). It is easy to see, however, that the above approach does not work as for $\epsilon \neq 0$ the Lax pair (64) *does not define a differential equation*. Indeed, note that (64) is equivalent to the Lax pair

$$(66) \quad \frac{dA^0}{dt} = [A^0, B^0] - \frac{\epsilon h}{I_1} \begin{pmatrix} 0 & y & i\bar{y} \\ -\bar{y} & i\gamma_3 & 0 \\ iy & 0 & -i\gamma_3 \end{pmatrix}.$$

Its (1, 2) entry is computed to be

$$\frac{d\beta}{dt} = \frac{i}{I_1} (yI_3\Omega_3 - x\gamma_3 + hz_0I_1y) - \frac{\epsilon h y}{I_1}$$

and the (3, 1) entry is

$$i \frac{d\beta}{dt} = \frac{1}{I_1} (-yI_3\Omega_3 + x\gamma_3 - hz_0I_1y) + \frac{\epsilon^3 h y}{I_1},$$

so $y \equiv 0$ and in a similar way $\bar{y} \equiv 0$.

More generally, it is seen from the coefficients of the spectral curve X_ϵ , $\epsilon \neq 0$, that the functions

$$\Omega_1^2 + \Omega_2^2, \quad \gamma_1^2 + \gamma_2^2, \quad \Omega_1\gamma_1 + \Omega_2\gamma_2, \quad \gamma_3, \quad \Omega_3$$

are invariants for *any* isospectral deformation of the matrix A^ϵ . By continuity these five functions are invariants for $\epsilon = 0$ too, so the vector field in \mathbf{C}^6 obtained as $\epsilon \rightarrow 0$ is collinear to the linear vector field of (3). Of course there is no analytic change of variables in \mathbf{C}^6 which sends the orbits of (3) to orbits of (2).

REFERENCES

- [1] ADLER, M. and P. VAN MOERBEKE. Linearization of Hamiltonian systems, Jacobi varieties and representation theory. *Advances in Math.* 38 (1980), 318–379.
- [2] AUDIN, M. and R. SILHOL. Variétés abéliennes réelles et toupie de Kowalevski. *Compositio Math.* 87 (1993), 153–229.
- [3] AUDIN, M. *Spinning Tops*. Cambridge Studies in Advanced Mathematics 51, Cambridge, 1996.
- [4] BATEMAN MANUSCRIPT PROJECT, A. Erdély (ed.). *Higher Transcendental Functions*, vol. II. McGraw-Hill, 1953.

- [5] BELOKOLOS, E. D., A. I. BOBENKO, V. Z. ENOL'SKIĬ, A. R. ITS and V. B. MATVVEEV. *Algebro-Geometric Approach to Nonlinear Integrable Equations*. Springer, 1994.
- [6] BEAUVILLE, A. Jacobiennes des courbes spectrales et systèmes hamiltoniens complètement intégrables. *Acta Math.* 164 (1990), 211–235.
- [7] DUBROVIN, B. A. Theta functions and non-linear equations. *Russ. Math. Surv.* 36, No. 2 (1981), 11–92 .
- [8] ——— Matrix finite zone operators. *J. Sov. Math.* 28 (1985), 20–50.
- [9] DUBROVIN, B. A., I. M. KRICHEVER and S. P. NOVIKOV. Integrable Systems, I. *Dynamical Systems IV*, 173–280. *Encyclopaedia of Mathematical Sciences, vol. 4* (V. I. Arnold, S. P. Novikov, eds.). Springer, 1990.
- [10] FAY, J. *Theta Functions on Riemann Surfaces*. Lect. Notes in Mathematics, no. 352. Springer, 1973.
- [11] GAVRILOV, L. and A. ZHIVKOV. The complex geometry of the Lagrange top. Preprint no. 61, Laboratoire Emile Picard, Université de Toulouse III, 1995.
- [12] GAVRILOV, L. Generalized Jacobians of spectral curves and completely integrable systems. *Math. Zeitschrift*, to appear.
- [13] GRIFFITHS, P. and J. HARRIS. *Principles of Algebraic Geometry*. J. Wiley and Sons, 1978.
- [14] JACOBI, C. Sur la rotation d'un corps. *Gesammelte Werke, Bd 2*, 289–352. Chelsea, 1969.
- [15] ——— Fragments sur la rotation d'un corps tirés des manuscrits de Jacobi et communiqués par E. Lotner. *Gesammelte Werke, Bd 2*, 425–514. Chelsea, 1969.
- [16] KLEIN, F. and A. SOMMERFELD. *Theorie des Kreisels*. Teubner, Leipzig, 1897–1910.
- [17] LAGRANGE, J. L. *Mécanique Analytique*, 1788. *Œuvres de Lagrange, tome XII*. Gauthier-Villars, 1889.
- [18] MUMFORD, D. *Tata Lectures on Theta II*. Progress in Mathematics, vol. 43. Birkhäuser, 1984.
- [19] POISSON, D. Sur un cas particulier du mouvement de rotation des corps pesans. *J. de l'École Polytechnique, tome IX, seizième cahier*, 1813.
- [20] PREVIATO, E. Hyperelliptic quasi-periodic and soliton solutions of the nonlinear Schrödinger equation. *Duke Math. J.* 52 (1985), 329–377.
- [21] RATIU, T. and P. VAN MOERBEKE. The Lagrange rigid body motion. *Ann. Inst. Fourier (Grenoble)* 32 (1982), 211–234.
- [22] RATIU, T. Euler-Poisson equations on Lie algebras and the n -dimensional heavy rigid body. *Amer. J. of Math.* 104 (1982) 409–448 .
- [23] SERRE, J.-P. *Groupes algébriques et corps de classes*. Hermann, 1959.
- [24] VERDIER, J.-L. Algèbres de Lie, systèmes hamiltoniens, courbes algébriques. *Séminaire E.N.S. (1979–82)*. Progress in Mathematics, vol. 37. Birkhäuser (1983), 237–246.
- [25] WEIL, A. Remarques sur un mémoire d'Hermite. *Collected Papers, vol. 2*, 111–116.

- [26] WHITTAKER, E.T. *A Treatise on the Analytical Dynamics of Particles and Bodies*. Cambridge Univ. Press, 1904.

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