Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 45 (1999)

Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: HARMONIC ANALYSIS ON VECTOR BUNDLES OVER Sp(1,n)/Sp(1)xSp(n)

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Kurzfassung

DOI: https://doi.org/10.5169/seals-64447

L'Enseignement Mathématique, t. 45 (1999), p. 219–252

HARMONIC ANALYSIS ON VECTOR BUNDLES OVER $\mathrm{Sp}(1,n)/\mathrm{Sp}(1)\times\mathrm{Sp}(n)$

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ABSTRACT. Harmonic analysis on vector bundles over $\mathrm{Sp}(1,n)/\mathrm{Sp}(1)\times\mathrm{Sp}(n)$ associated with a finite dimensional representation τ of $\mathrm{Sp}(1)$ is developed using Godement's approach of trace spherical functions. The trace spherical trace functions are written in terms of Jacobi functions, and among them the positive definite ones are singled out. An inversion formula for the generalized Abel transform is given explicitly. The Paley-Wiener theorem, the inversion formula and the Plancherel theorem for the τ-spherical transform are determined.

INTRODUCTION

Harmonic analysis over Riemannian symmetric spaces of noncompact type is a fundamental and powerful area of mathematics that exhibits a beautiful interplay between the theory of special functions and the representation theory of semisimple Lie groups. Grown around the monumental work of Harish-Chandra, it has nowadays reached a nearly complete formulation, but, in its development, it has also laid the foundations of its natural extension: harmonic analysis on vector bundles over Riemannian symmetric spaces of noncompact type. Motivated also by many physical applications, this new subject is currently studied very intensively (cf. for instance [BR], [O], [Shi], [Cam], [P], [vdV], [M], [Dei], [BOS]), but a general theory has not yet been formulated.

In this paper we present a complete treatment of harmonic analysis for the spherical transform on a certain class of vector bundles over the hyperbolic space $\mathrm{Sp}(1,n)/\mathrm{Sp}(1)\times\mathrm{Sp}(n)$. Set $G=\mathrm{Sp}(1,n)$ and $K=\mathrm{Sp}(1)\times\mathrm{Sp}(n)$. The class of vector bundles we consider are those associated with finite-dimensional irreducible representations τ of K which are trivial on $\mathrm{Sp}(n)$, so actually finite dimensional representations of $\mathrm{Sp}(1)\cong\mathrm{SU}(2)$. This setting is sufficiently