

<b>Zeitschrift:</b>	L'Enseignement Mathématique
<b>Herausgeber:</b>	Commission Internationale de l'Enseignement Mathématique
<b>Band:</b>	45 (1999)
<b>Heft:</b>	3-4: L'ENSEIGNEMENT MATHÉMATIQUE
 <b>Artikel:</b>	PRODUCT MEASURABILITY, PARAMETER INTEGRALS, AND A FUBINI COUNTEREXAMPLE
<b>Autor:</b>	Mattner, Lutz
<b>Kapitel:</b>	2.2 A FUBINI COUNTEREXAMPLE
<b>DOI:</b>	<a href="https://doi.org/10.5169/seals-64449">https://doi.org/10.5169/seals-64449</a>

### Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Siehe Rechtliche Hinweise.

### Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. Voir Informations légales.

### Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. See Legal notice.

**Download PDF:** 30.03.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

## 2.2 A FUBINI COUNTEREXAMPLE

In this section, we give an example of (2). Let  $\mathcal{A}$  be as in (5) and define  $\mu|_{\mathcal{A}}$  by

$$(11) \quad \mu(A) := \begin{cases} 0 & \text{if } A \text{ is meager,} \\ 1 & \text{if } A \text{ is comeager.} \end{cases}$$

This is possible, since no set  $A \subset \mathbf{R}$  is simultaneously meager and comeager, for otherwise  $\emptyset = A \cap A^c$  would be comeager, in contradiction to Baire's theorem. It is easy to check that  $\mu$  is a probability measure on  $(\mathbf{R}, \mathcal{A})$ . Let again  $\nu := \lambda :=$  Lebesgue measure on  $\mathcal{B} := \mathcal{B}(\mathbf{R})$ , and choose  $A \in \mathcal{A}$  meager with  $\lambda(A^c) = 0$ . Then  $1_A(\cdot + y)$  is  $\mathcal{A}$ -measurable with

$$\int_{\mathbf{R}} 1_A(x + y) d\mu(x) = \mu(A - y) = 0 \quad (y \in \mathbf{R}).$$

On the other hand, we have

$$\int_{\mathbf{R}} 1_A(x + y) d\nu(y) = \lambda(A - x) = \infty \quad (x \in \mathbf{R}).$$

Hence (2) is obviously true in this case.

## 3. MEASURABILITY

Here is a positive result, having a certain measurability property of  $F$  from (1) among its conclusions. An application of this occurs in Mattner (1999).

**3.1. THEOREM.** *Let  $(\mathcal{X}, \mathcal{A}, \mu)$  and  $(\mathcal{Y}, \mathcal{B}, \nu)$  be  $\sigma$ -finite measure spaces, let  $f: \mathcal{X} \times \mathcal{Y} \rightarrow [0, \infty]$  be a function measurable with respect to the product  $\sigma$ -algebra  $\mathcal{A} \otimes \mathcal{B}$ , and put*

$$\mathcal{A}_0 := \sigma(\{f(\cdot, y) : y \in \mathcal{Y}\}),$$

$$\mathcal{B}_0 := \sigma(\{f(x, \cdot) : x \in \mathcal{X}\}),$$

$$\bar{\mathcal{A}}_0 := \{A \in \mathcal{A} : \exists A_0 \in \mathcal{A}_0 \text{ with } A = A_0 \text{ } [\mu]\},$$

$$\bar{\mathcal{B}}_0 := \{B \in \mathcal{B} : \exists B_0 \in \mathcal{B}_0 \text{ with } B = B_0 \text{ } [\nu]\},$$

$$\overline{\mathcal{A}_0 \otimes \mathcal{B}_0} := \{C \in \mathcal{A} \otimes \mathcal{B} : \exists C_0 \in \mathcal{A}_0 \otimes \mathcal{B}_0 \text{ with } C = C_0 \text{ } [\mu \otimes \nu]\}.$$

*Then  $f$  is  $\overline{\mathcal{A}_0 \otimes \mathcal{B}_0}$ -measurable,  $\int_{\mathcal{Y}} f(\cdot, y) d\nu(y)$  is  $\bar{\mathcal{A}}_0$ -measurable, and  $\int_{\mathcal{X}} f(x, \cdot) d\mu(x)$  is  $\bar{\mathcal{B}}_0$ -measurable.*