

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 45 (1999)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: PRODUCT MEASURABILITY, PARAMETER INTEGRALS, AND A FUBINI COUNTEREXAMPLE
Autor: Mattner, Lutz

Bibliographie
DOI: <https://doi.org/10.5169/seals-64449>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 14.03.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

$$\int_{B_0} f(\cdot, y) d\bar{\nu}_0(y) = \int_{B_0} \tilde{f}(\cdot, y) d\bar{\nu}_0(y) \quad [\bar{\mu}_0].$$

Trivially, this remains true if $[\bar{\mu}_0]$ is replaced by $[\mu]$, and an integration yields

$$(18) \quad \int_A \int_{B_0} f(x, y) d\bar{\nu}_0(y) d\mu(x) = \int_A \int_{B_0} \tilde{f}(x, y) d\bar{\nu}_0(y) d\mu(x)$$

($A \in \mathcal{A}$, $B_0 \in \bar{\mathcal{B}}_0$). We now want to interchange the order of integrations. Since \tilde{f} is trivially $\mathcal{A} \otimes \bar{\mathcal{B}}_0$ -measurable, we may obviously do this on the right hand side of (18). To do the same on the left hand side, we rewrite it successively as

$$\int_A \int_{B_0} f(x, y) d\nu(y) d\mu(x) = \int_{B_0} \int_A f(x, y) d\mu(x) d\nu(y) = \int_{B_0} \int_A f(x, y) d\mu(x) d\bar{\nu}_0(y),$$

where the last equality follows from a second application of Claim 1, with the role of the variables interchanged. Thus (18) yields

$$(19) \quad \int_{B_0} \int_A f(x, y) d\mu(x) d\bar{\nu}_0(y) = \int_{B_0} \int_A \tilde{f}(x, y) d\mu(x) d\bar{\nu}_0(y)$$

($A \in \mathcal{A}$, $B_0 \in \bar{\mathcal{B}}_0$). Now the argument leading from (17) to (18) can be repeated to lead from (19) to a corresponding statement with B in place of B_0 , ν in place of $\bar{\nu}_0$, and \mathcal{B} in place of $\bar{\mathcal{B}}_0$, which is equivalent to

$$\int_{A \times B} f d\mu \otimes \nu = \int_{A \times B} \tilde{f} d\mu \otimes \nu \quad (A \in \mathcal{A}, B \in \mathcal{B}).$$

This shows that $f = \tilde{f}$ $[\mu \otimes \nu]$, which yields the desired conclusion. \square

ACKNOWLEDGEMENT. The present work was supported by a Heisenberg grant of the Deutsche Forschungsgemeinschaft. I thank H. von Weizsäcker for helpful remarks on an earlier version.

REFERENCES

- COHN, D.L. (1980) *Measure Theory*. Birkhäuser.
 EDWARDS, R.E. (1965) *Functional Analysis. Theory and Applications*. Holt, Rinehart and Winston.
 ELSTRODT, J. (1996) *Maß- und Integrationstheorie*. Springer.
 FRIEDMAN, H. (1980) A consistent Fubini-Tonelli theorem for nonmeasurable functions. *Illinois J. Math.* 24, 390–395.

- MATTNER, L. (1999) Minimal sufficient statistics in location-scale parameter models.
[Submitted for publication.]
- OXTOBY, J. C. (1980) *Measure and Category*. 2nd ed. Springer, New York.
- ROYDEN, H. L. (1988) *Real Analysis*. 3rd ed. Prentice Hall, Englewood Cliffs.
- RUDIN, W. (1987) *Real and Complex Analysis*. 3rd ed. McGraw-Hill.
- RUDIN, W. (1991) *Functional Analysis*. 2nd ed. McGraw-Hill.
- SIERPIŃSKI, W. (1920) Sur les rapport entre l'existence des intégrales $\int_0^1 f(x, y) dx$, $\int_0^1 f(x, y) dy$ et $\int_0^1 dx \int_0^1 f(x, y) dy$. *Fund. Math.* 1, 142–147. Reprinted in: Sierpiński (1975), pages 341–345.
- SIERPIŃSKI, W. (1975) *Œuvres Choisies, Tome II*. Państwowe Wydawnictwo Naukowe, Warszawa.

(Reçu le 2 mars 1999)

Lutz Mattner

Universität Hamburg
Institut für Mathematische Stochastik
Bundesstr. 55
D-20146 Hamburg
Germany
e-mail: mattner@math.uni-hamburg.de

Vide-leer-empty