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## ON GROUPS ACTING ON NONPOSITIVELY CURVED CUBICAL COMPLEXES

by Werner BALLMANN<sup>1)</sup> and Jacek ŚWIĄTKOWSKI<sup>2)</sup>

ABSTRACT. We study groups acting on simply connected cubical complexes of nonpositive curvature. Our main objectives are related actions on trees, the existence of free subgroups and the existence of homomorphisms onto free abelian groups.

### INTRODUCTION

We study groups acting on simply connected cubical complexes of nonpositive curvature. Examples of such groups and spaces arise naturally from many constructions. Among them are graph products of groups and other groups acting on right-angled buildings, fundamental groups of hyperbolizations of polyhedra, of toric manifolds and of blow-ups of arrangements of hyperplanes, and many others (see [Da], [DJ1], [DJ2], [DJS] and Section 2 below). Roughly speaking, a *cubical complex* is a cell complex whose cells are cubes. As a definition of nonpositive curvature we use the comparison triangle condition  $CAT(0)$  with respect to the natural *cubical metric* of a cubical complex (see Section 1 below for more details).

It turns out that groups acting on nonpositively curved cubical complexes share many properties with groups acting on trees and with infinite Coxeter groups. For example, if  $\Gamma$  is a group satisfying Property (T), then any automorphic action of  $\Gamma$  on a tree, a Coxeter complex, a Euclidean space or a hyperbolic space has a fixed point, see [HV], Chapter 6. The same result holds for actions of  $\Gamma$  on cubical complexes, a result recently proved by Niblo and Reeves, see [NR]. This result and our related results in [BS] are the source of our interest in cubical complexes.

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