

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 45 (1999)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: COUNTING PATHS IN GRAPHS
Autor: Bartholdi, Laurent
Kapitel: 8.1 QUASI-FREE
DOI: <https://doi.org/10.5169/seals-64442>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 30.03.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

8.1 Π QUASI-FREE

Let S, T be finite sets, and $\bar{\cdot}$ an involution on S . Consider the two presentations

$$\begin{aligned}\Pi &= \langle S \mid s\bar{s} = 1 \quad \forall s \in S \rangle, \\ \Pi &= \langle S \cup T \mid s\bar{s} = 1 \quad \forall s \in S; t = 1 \quad \forall t \in T \rangle.\end{aligned}$$

Let $\Xi < \Pi$ be any subgroup, and let F' and G' be the generating series related to the first presentation. Clearly $F' = F$, as both series count the same objects in Π (regardless of Π 's presentation); while

$$G(t) = \frac{G'\left(\frac{t}{1-|T|t}\right)}{1-|T|t}.$$

Indeed any word $w = w_1 \dots w_n$ in $S \cup T$ defining an element of Ξ can be uniquely decomposed as $w = t_0 s_1 t_1 \dots s_m t_m$, where $s_i \in S$, t_i are words in T for all i , and $s_1 \dots s_m$ defines an element of Ξ ; moreover all choices of $s_1 \dots s_m$ defining an element of Ξ and words t_i in T give a distinct word w . It then suffices to note that the generating series for any of the t_i is $1/(1-|T|t)$.

Putting everything together, we obtain:

PROPOSITION 8.1. *Let Π be as above, $\Xi < \Pi$ a subgroup. Then*

$$\frac{F(t)}{1-t^2} = \frac{G\left(\frac{t}{1+|T|t+(|S|-1)t^2}\right)}{1+|T|t+(|S|-1)t^2}.$$

8.2 $\Pi = \mathbf{PSL}_2(\mathbf{Z})$

Let

$$\Pi = \mathbf{PSL}_2(\mathbf{Z}) = \langle a, b \mid a^2, b^3 \rangle,$$

and let $\Xi < \Pi$ be any subgroup. We take $S = \{a, b, b^{-1}\}$.

We suppose Ξ is torsion-free, i.e. contains no element of the form waw^{-1} or $wb^{\pm 1}w^{-1}$. Let \mathcal{X} be the Schreier graph of $(\Pi, \{a, b, b^{-1}\})$ relative to Ξ , as defined in Subsection 3.1; it is a trivalent graph whose vertex set is $\Xi \backslash \Pi$. Its vertices can be grouped in triples $w^\Delta = \{w, wb, wb^{-1}\}$ connected in triangles. Let \mathcal{F} be the graph obtained from \mathcal{X} by identifying each triple to a vertex. Explicitly,