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$P(2)$  into 8 triangles  $D_j$  so that  $a_j$  is a side of  $D_j$ ,  $j = 1, \dots, 8$ , compare Figure 7. Since  $M$  is hyperelliptic,  $D_j$  and  $D_{j+4}$  are isometric,  $j = 1, \dots, 4$ . Denote by  $\delta_i$  the angle of  $D_i$  in the vertex  $C$ ,  $i = 1, \dots, 4$ . The seven lengths determine the triangles  $D_i$ ,  $i = 1, 2, 3$ , as well as two sides and the angle  $\delta_4$  of  $D_4$  by the condition

$$(6) \quad \Delta := \sum_{j=1}^4 \delta_j = \pi,$$

so they determine also  $D_4$ . This shows that the seven lengths determine  $P(2)$ . Multiply the seven lengths by a positive real  $t$  and assume that the seven new lengths also determine a canonical polygon  $P_t(2)$ . If  $t > 1$ , then  $\delta_i$ ,  $i = 1, 2, 3$ , are smaller in  $P_t(2)$  than in  $P(2)$  by Lemma 9, therefore, by (6),  $\delta_4$  is larger in  $P_t(2)$  than in  $P(2)$ . It follows by Lemma 7 that the sum of the two other angles of  $D_4$  is smaller in  $P_t(2)$  than in  $P(2)$ . Since all angles in  $D_i$ ,  $i = 1, 2, 3$ , are smaller in  $P_t(2)$  than in  $P(2)$  by Lemma 9, it follows that

$$\sum_{i=1}^4 \alpha_i$$

is smaller in  $P_t(2)$  than in  $P(2)$ . But this contradicts condition (II) of canonical polygons. An analogous contradiction follows if  $t < 1$  proving thus that  $t = 1$  and therefore the theorem.  $\square$

REMARK. Theorem 16 is new. It is well known that  $6g-6$  length functions can never parametrize  $T_g$  so that the situation of Theorem 16 is the best we can expect. It is not known whether  $6g-5$  geodesic length functions, *taken as homogeneous parameters*, can parametrize  $T_g$  for  $g \geq 3$ .

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