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Artikel: TEICHMÜLLER SPACE AND FUNDAMENTAL DOMAINS OF FUCHSIAN GROUPS

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$P(2)$ into 8 triangles D_j so that a_j is a side of D_j, $j = 1, \dots, 8$, compare Figure 7. Since M is hyperelliptic, D_j and D_{j+4} are isometric, $j = 1, \dots, 4$. Denote by δ_i the angle of D_i in the vertex C, $i = 1, \dots, 4$. The seven lengths determine the triangles D_i, $i = 1, 2, 3$, as well as two sides and the angle δ_4 of D_4 by the condition

$$\Delta := \sum_{j=1}^{4} \delta_j = \pi , \tag{6}$$

so they determine also D_4. This shows that the seven lengths determine $P(2)$. Multiply the seven lengths by a positive real t and assume that the seven new lengths also determine a canonical polygon $P_t(2)$. If $t > 1$, then δ_i, $i = 1, 2, 3$, are smaller in $P_t(2)$ than in $P(2)$ by Lemma 9, therefore, by (6), δ_4 is larger in $P_t(2)$ than in $P(2)$. It follows by Lemma 7 that the sum of the two other angles of D_4 is smaller in $P_t(2)$ than in $P(2)$. Since all angles in D_i, $i = 1, 2, 3$, are smaller in $P_t(2)$ than in $P(2)$ by Lemma 9, it follows that

$$\sum_{i=1}^{4} \alpha_i$$

is smaller in $P_t(2)$ than in $P(2)$. But this contradicts condition (II) of canonical polygons. An analogous contradiction follows if $t < 1$ proving thus that $t = 1$ and therefore the theorem. □

REMARK. Theorem 16 is new. It is well known that $6g-6$ length functions can never parametrize T_g so that the situation of Theorem 16 is the best we can expect. It is not known whether $6g - 5$ geodesic length functions, *taken as homogeneous parameters*, can parametrize T_g for $g \geq 3$.

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