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ON THE CONSTRUCTION OF GENERALIZED JACOBIANS

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ABSTRACT. We give a modern exposition of the construction of generalized jacobians using Weil's method.

0. INTRODUCTION

Generalized jacobians of algebraic curves are treated in detail in [S]. In this book Serre uses the terminology "generic points" that is developed in Weil's *Foundations of Algebraic Geometry*. Nowadays one uses the terminology in Grothendieck's *Éléments de Géométrie Algébrique*, and it is hard for students studying algebraic geometry to get used to Weil's terminology. At least my personal experience tells me so. So in this paper we use Weil's method and Grothendieck's language to construct generalized jacobians.

In §1 we state a theorem of Grothendieck that is used throughout this paper. In §2 we list some basic properties of relative effective Cartier divisors. We construct a birational group in §3 and show how to get an algebraic group from a birational group in §4. In §5 we prove some fundamental properties of generalized jacobians. The main results are Theorems 1 and 2. In §6 we prove that the generalized jacobian of a curve is the Picard scheme of the curve. The Appendix contains the proof of a technical proposition.

While preparing this note, I was helped by [A], [BLR], [Mi] and [S].