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Artikel: IDEAL SOLUTIONS OF THE TARRY-ESCOTT PROBLEM OF DEGREE FOUR AND A RELATED DIOPHANTINE SYSTEM

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Kapitel: 4. The diophantine system $\sum_{i=1}^5 a_i^r = \sum_{i=1}^5 b_i^5, \quad r=1,2,3,4,6$

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We may apply Frolov's theorem to the above solution to obtain other non-symmetric solutions. For instance, an arbitrary constant K can be added to all the terms a_i, b_i, $i = 1, 2, 3, 4, 5$.

As a numerical example, taking $m = 3$, $n = 1$, we get, on suitable re-arrangement, the following solution:

$$(-1659)^r + 1406^r + 2784^r + 4025^r + 5915^r$$
$$= (-1675)^r + 1659^r + 2366^r + 4256^r + 5865^r,$$

where $r = 1, 2, 3, 4$. Adding the constant 1676 to all the terms, we get the following solution in positive integers:

$$17^r + 3082^r + 4460^r + 5701^r + 7591^r = 1^r + 3335^r + 4042^r + 5932^r + 7541^r,$$

where $r = 1, 2, 3, 4$.

4. The diophantine system $\sum_{i=1}^5 a_i^r = \sum_{i=1}^5 b_i^r$, $r = 1, 2, 3, 4, 6$

We will now state the theorem given by Gloden [7, p. 24] to which a reference has already been made in the introduction and then apply it to obtain a parametric solution of this diophantine system.

THEOREM 4.1. *If*

$$\sum_{i=1}^{k+1} a_i^r = \sum_{i=1}^{k+1} b_i^r, \qquad r = 1, 2, \ldots, k$$

then

$$\sum_{i=1}^{k+1} (a_i + t)^r = \sum_{i=1}^{k+1} (b_i + t)^r, \qquad r = 1, 2, \ldots, k, k+2,$$

where

$$t = -\Big(\sum_{i=1}^{k+1} a_i\Big)/(k+1).$$

As we have already obtained, in the preceding section, a parametric solution of $\sum_{i=1}^5 a_i^r = \sum_{i=1}^5 b_i^r$, $r = 1, 2, 3, 4$, a direct application of the above theorem gives a parametric solution of $\sum_{i=1}^5 a_i^r = \sum_{i=1}^5 b_i^r$, $r = 1, 2, 3, 4$ and 6. We multiply the $(a_i + t)$, $(b_i + t)$, $i = 1, 2, 3, 4, 5$ by 5 to cancel out denominators, and we rename the resulting expressions as a_i, b_i, $i = 1, 2, 3, 4, 5$, so that the parametric solution of the diophantine system

$$\sum_{i=1}^{5} a_i^r = \sum_{i=1}^{5} b_i^r, \qquad r = 1, 2, 3, 4, 6$$

may be written as

$$\begin{aligned}
a_1 &= 4m^8 - 30m^7n + 98m^6n^2 - 34m^5n^3 - 9m^4n^4 \\
&\quad - 80m^3n^5 + 97m^2n^6 - 6mn^7 - 16n^8, \\
a_2 &= 4m^8 - 20m^7n + 63m^6n^2 - 184m^5n^3 + 116m^4n^4 \\
&\quad + 60m^3n^5 - 53m^2n^6 - 26mn^7 + 24n^8, \\
a_3 &= -16m^8 + 53m^6n^2 + 46m^5n^3 - 279m^4n^4 \\
&\quad + 460m^3n^5 - 388m^2n^6 + 144mn^7 - 16n^8, \\
a_4 &= -16m^8 + 90m^7n - 262m^6n^2 + 361m^5n^3 - 279m^4n^4 \\
&\quad + 145m^3n^5 - 73m^2n^6 + 54mn^7 - 16n^8, \\
a_5 &= 24m^8 - 40m^7n + 48m^6n^2 - 189m^5n^3 + 451m^4n^4 \\
&\quad - 585m^3n^5 + 417m^2n^6 - 166mn^7 + 24n^8, \\
b_1 &= 24m^8 - 40m^7n + 23m^6n^2 + 36m^5n^3 - 284m^4n^4 \\
&\quad + 520m^3n^5 - 393m^2n^6 + 134mn^7 - 16n^8, \\
b_2 &= -16m^8 - 10m^7n + 138m^6n^2 - 254m^5n^3 + 391m^4n^4 \\
&\quad - 540m^3n^5 + 437m^2n^6 - 166mn^7 + 24n^8, \\
b_3 &= 4m^8 - 70m^7n + 188m^6n^2 - 229m^5n^3 + 121m^4n^4 \\
&\quad + 55m^3n^5 - 143m^2n^6 + 74mn^7 - 16n^8, \\
b_4 &= 4m^8 + 20m^7n - 127m^6n^2 + 86m^5n^3 + 121m^4n^4 \\
&\quad - 260m^3n^5 + 172m^2n^6 - 16mn^7 - 16n^8, \\
b_5 &= -16m^8 + 100m^7n - 222m^6n^2 + 361m^5n^3 - 349m^4n^4 \\
&\quad + 225m^3n^5 - 73m^2n^6 - 26mn^7 + 24n^8.
\end{aligned}$$

As a numerical example, when $m = 3$ and $n = 1$, we get, on removal of common factors and suitable re-arrangement, the following solution:

$$\begin{aligned}
&1449^r + 7654^r + 17104^r + (-5441)^r + (-20766)^r \\
&\qquad = 8809^r + 16854^r + (-641)^r + (-4176)^r + (-20846)^r,
\end{aligned}$$

where $r = 1, 2, 3, 4$ and 6.