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THE WITT GROUP OF LAURENT POLYNOMIALS

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ABSTRACT. We give a direct, self-contained proof of the fact that for a large class of rings A, in particular for all regular rings with involution, $W(A[t, 1/t]) = W(A) \oplus W(A)$.

1. INTRODUCTION

The purpose of this note is to give a short direct proof of two fundamental theorems on the Witt group of polynomials and Laurent extensions of a ring A. These theorems were proved independently by M. Karoubi [3] and by A. Ranicki [5]. We will state them under the most general conditions on A and for their proofs we will use nothing more than a general result on the K-theory of Laurent polynomials. In the last section we will show, by two counterexamples, that the assumptions we make on A are necessary.

We begin by recalling briefly some definitions. We refer to [4] for a more detailed exposition and for the proofs of the few basic results that we will use.

Let A be an associative ring with an involution denoted by $a \mapsto a^\circ$. Except in §2 we will always assume that 2 is invertible in A. If M is a right A-module, we denote by M^* its dual $\mathrm{Hom}_A(M, A)$ endowed with the right action of A given by $fa(x) = a^\circ f(x)$ for any $f \colon M \to A$ and $a \in A$. If P is a finitely generated projective right A-module we identify it with P^{**} through the canonical isomorphism mapping $x \in P$ to $\widehat{x} \colon P^* \to A$ defined by $\widehat{x}(f) = f(x)$.

Let ϵ be 1 or -1. An *ϵ-hermitian space* over A is a pair (P, α) consisting of a finitely generated projective right A-module P and an A-isomorphism $\alpha \colon P \to P^*$ satisfying $\alpha = \epsilon\alpha^*$. For brevity ϵ-hermitian spaces will be called *spaces*. A 1-hermitian space (over a commutative ring A) is also called a *quadratic space*.