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THE SIXTH FERMAT NUMBER
AND PALINDROMIC CONTINUED FRACTIONS

by Freeman DYSON

ABSTRACT. An elementary argument is presented, verifying the known factorization of the sixth Fermat number $2^{64} + 1$. The verification is made more elegant by using an old theorem of Serret concerning palindromic continued fractions.

It is easy to explain why the fifth Fermat number $2^{32} + 1$ is divisible by 641 [Hardy and Wright, 1938]. The prime 641 has the two representations

$$(1) \quad 641 = 5 \cdot 2^7 + 1 = 5^4 + 2^4.$$

The congruence

$$(2) \quad 2^{32} = 2^4 \cdot 2^{28} \equiv -5^4 \cdot 2^{28} = -(5 \cdot 2^7)^4 \equiv -1 \pmod{641}$$

follows immediately from (1).

The purpose of this note is to explain in a similarly elementary way why the sixth Fermat number $2^{64} + 1$ is divisible by 274177. The divisor has the representation

$$(3) \quad q = 274177 = 1 + 2^8 f, \quad f = (2^6 - 1)(2^4 + 1),$$

and it is easily verified that

$$(4) \quad 2^{24} - 1 = fg, \quad g = (2^6 + 1)(2^8 - 2^4 + 1).$$

We look for a factorization of the form

$$(5) \quad 2^{64} + 1 = (x^2 + y^2)(z^2 + w^2), \quad 2^{32} - i = (x + iy)(z - iw),$$

so that we require integers x, y, z, w satisfying

$$(6) \quad xz + yw = 2^{32}, \quad xw - yz = 1, \quad x^2 + y^2 = q.$$

When $z = gx$, $w = gy$, the right side of (5), $(x + iy)g(x - iy)$, becomes gq ; and $gq = 2^{32} + a$, very close to 2^{32} , with the difference $a = g - 2^8 = 15409$