Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 46 (2000)

Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THREE REMARKS ON GEODESIC DYNAMICS AND FUNDAMENTAL GROUP

Autor: Gromov, Mikhaïl

Bibliographie

DOI: https://doi.org/10.5169/seals-64809

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Siehe Rechtliche Hinweise.

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. Voir Informations légales.

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. See Legal notice.

Download PDF: 02.04.2025

ETH-Bibliothek Zürich, E-Periodica, https://www.e-periodica.ch

3. In order to make use of π_1 in locating other (not locally minimal) closed geodesics without non-degeneracy condition, one has to extend [Gr] to the non simply connected situation. When V is homeomorphic to $V_0 \times S^1$ and V_0 is simply connected, we can apply [Gr] directly and get $N_m(V) \geq C \log(m)$ for some $C > 0$. (Probably this is true when $H_1(V)$ is infinite or at least when $\pi_1(V) = \mathbf{Z}$.) The last estimate can be sharpened and we show this here for the simplest example when V_0 is the sphere S^3 and the proof is obvious [Gr].

Let V be homeomorphic to $S^3 \times S^1$. Then there exist closed geodesics $g_j^i \subset V$ (not necessarily simple) such that

1. *Each g_j^i, $i,j = 1,2,\ldots$, represents $\gamma^i \in \pi_1(V)$ where γ is a generator in $\pi_1(V)$.*
2. *For each i the geodesic g_1^i is the shortest in its homotopy class.*

Denote by $|g_j^i|$ the length of g_j^i.

3. $|g_1^{i+k}| + C \geq |g_1^i| + |g_1^k| \geq |g_1^{i+k}|$, *where $C \geq 0$ and $i,k = 1,2,\ldots$*
4. $|g_j^i| + C \geq |g_{j+1}^i| \geq |g_j^i|$ *for some $C > 0$ and $i,j = 1,2,\ldots$*
5. $\left||g_j^i| - |g_j^k|\right| \leq C|i-k|$ *for some $C > 0$ and $i,j,k = 1,2,\ldots$*
6. $|g_j^i| \geq \frac{i}{C}$ *for some $C > 0$ and $i,j = 1,2,\ldots$*

COROLLARY. *If V is as above, then $\limsup_{m\to\infty} \frac{N_m(V)}{m^2} \geq const > 0$.*

All our estimates give a rather poor approximation to the (unknown) reality. Probably, in most cases N_m grows exponentially. That is so, of course, for "C^0-generic" manifolds ("C^0-generic" is used for C^0-generic manifolds having uncountably many closed geodesics).

REFERENCES

[Fr] FRANKS, J. Anosov diffeomorphisms. In: *Global Analysis, Berkeley, 1968.* Proc. Symp. Pure Math. 14, Amer. Math. Soc. 1970, 61–93.

[G-M] GROMOLL, D. and W. MEYER. Periodic geodesics on compact manifolds. *J. Differential Geom. 3* (1969), 493–510.

[Gr] GROMOV, M. Homotopical effects of dilatation. *J. Differential Geom. 13* (1978), 303–310.

[Kl] KLINGENBERG, W. *Lectures on Closed Geodesics.* Third ed. Mathematisches Institut der Universität Bonn, 1977.

[Ma] MANNING, A. Topological entropy and the first homology group. *Springer Lecture Notes in Math. 468* (1974), 185–191.

[Nie] NIELSEN, J. Über topologische Abbildungen geschlossener Flächen. *Abh. Math. Sem. Univ. Hamburg 3* (1924), 246–260.

[Pu] PUGH, C. On the entropy conjecture. *Springer Lecture Notes in Math. 468* (1974), 257–262.

[Sh] SHUB, M. Endomorphisms of compact manifolds. *Amer. J. Math. 91* (1969), 185–199.

[Sh-S] SHUB, M. and D. SULLIVAN. A remark on the Lefschetz fixed point formula. *Topology 13* (1974), 189–191.

WHY THE APPENDICES WERE NOT WRITTEN: AUTHOR'S APOLOGIES TO THE READERS

APPENDIX 2. The stable homeomorphism suggests a geometric link between the homotopy and topological invariance of Pontryagin classes, at least for manifolds with negative curvature but I did not manage to forge this to my satisfaction till 1996 (see [Gro_2]); also see [Fa-Jo] for a deeper analysis.

APPENDIX 3. One can define a notion of hyperbolicity for an automorphism α of an arbitrary finitely generated group Γ, such that (Γ, α) functorially defines a Bowen-Franks hyperbolic system (see [Gro_1]). Unfortunately, this class of (Γ, α) is rather limited, e.g. is not closed under free products and does not include hyperbolic automorphisms of surface groups. I still do not know what the right setting is.

APPENDIX 4. An obvious example of semi-hyperbolicity is provided by non-strictly expanding endomorphisms, where the geometric picture is rather clear. However, I still do not see a functorial description, in the spirit of the symbolic dynamics, of more general semi-hyperbolic systems, not even for the geodesic (or Weil chamber) flows on locally symmetric spaces (compare [B-G-S] and [Br-Ha]).

APPENDIX 5. The section on entropy was inspired by Manning's paper [Ma_1], but I was unaware of the prior paper by Dinaburg (see [Din]) that essentially contained the entropy estimate for geodesic flows (also discussed in [Ma_2]). On the other hand, estimating the entropy of an endomorphism (or an automorphism) f in terms of $f_* : \pi_1 \to \pi_1$ appears now much less clear than it seemed to me back in 1976. It is not hard to bound the entropy from below