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Artikel: THREE REMARKS ON GEODESIC DYNAMICS AND FUNDAMENTAL GROUP

Autor: Gromov, Mikhaïl

Anhang: Why the Appendices were not written : author's apologies to the readers

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[Ma] MANNING, A. Topological entropy and the first homology group. *Springer Lecture Notes in Math. 468* (1974), 185–191.

[Nie] NIELSEN, J. Über topologische Abbildungen geschlossener Flächen. *Abh. Math. Sem. Univ. Hamburg 3* (1924), 246–260.

[Pu] PUGH, C. On the entropy conjecture. *Springer Lecture Notes in Math. 468* (1974), 257–262.

[Sh] SHUB, M. Endomorphisms of compact manifolds. *Amer. J. Math. 91* (1969), 185–199.

[Sh-S] SHUB, M. and D. SULLIVAN. A remark on the Lefschetz fixed point formula. *Topology 13* (1974), 189–191.

WHY THE APPENDICES WERE NOT WRITTEN: AUTHOR'S APOLOGIES TO THE READERS

APPENDIX 2. The stable homeomorphism suggests a geometric link between the homotopy and topological invariance of Pontryagin classes, at least for manifolds with negative curvature but I did not manage to forge this to my satisfaction till 1996 (see [Gro_2]); also see [Fa-Jo] for a deeper analysis.

APPENDIX 3. One can define a notion of hyperbolicity for an automorphism α of an arbitrary finitely generated group Γ, such that (Γ, α) functorially defines a Bowen-Franks hyperbolic system (see [Gro_1]). Unfortunately, this class of (Γ, α) is rather limited, e.g. is not closed under free products and does not include hyperbolic automorphisms of surface groups. I still do not know what the right setting is.

APPENDIX 4. An obvious example of semi-hyperbolicity is provided by non-strictly expanding endomorphisms, where the geometric picture is rather clear. However, I still do not see a functorial description, in the spirit of the symbolic dynamics, of more general semi-hyperbolic systems, not even for the geodesic (or Weil chamber) flows on locally symmetric spaces (compare [B-G-S] and [Br-Ha]).

APPENDIX 5. The section on entropy was inspired by Manning's paper [Ma_1], but I was unaware of the prior paper by Dinaburg (see [Din]) that essentially contained the entropy estimate for geodesic flows (also discussed in [Ma_2]). On the other hand, estimating the entropy of an endomorphism (or an automorphism) f in terms of $f_*: \pi_1 \to \pi_1$ appears now much less clear than it seemed to me back in 1976. It is not hard to bound the entropy from below via the "asymptotic stretch" of $f_*: \pi_1 \to \pi_1$ with respect to the word metric in π_1 (see [Bow]). But this is not sharp even for linear automorphisms of tori T^n, where the entropy is expressed by the "k-dimensional stretch" on H_1 for some $k \leq n$ that equals the spectral radius of f_* on H_k. Such k-stretch can be defined, in general, in terms of $f_*: \pi_1(S) \to \pi_1(S)$ and the classifying map $S \to K(\pi_1, 1)$ (refining the spectral radius of f_* on H^k coming from $K(\pi_1, 1)$), but my obvious "proof" of the lower bound on the entropy by this k-stretch missed a hidden trap. This was also overlooked in [Ma$_3$] (for $f_*: H_1 \to H_1$, where a proper identification of the "k-stretch" with the spectral radius needs extra work), as was pointed out to me much later by David Fried. (The difficulty already appears for closed subsets S in the torus T^n invariant under linear automorphisms f of T^n, where one wishes to estimate the entropy of $f|S$ in terms of f_* acting on the *spectral* cohomology of S coming from T^n. On the other hand, the case of $T^n \to T^n$ is settled in [Mi-Pr].)

APPENDIX 6. Probably, the recent progress in Nielsen theory allows a description of the cases, where card(Fix$f$) is well controlled from below by some twisted Lefschetz number (see [Fel]).

APPENDIX 7. Nothing interesting to say.

APPENDIX 8. Minima of geometric functionals related to the logical complexity have been studied in depth by A. Nabutovski (see [Na] and references therein). Yet I do not feel ready yet to write this Appendix. For example, I do not see what is the actual influence of a suitable (?) complexity measure of $\pi_1(V)$ on the Plateau problem in V.

BIBLIOGRAPHY

[B-G-S] BALLMANN, W., M. GROMOV and V. SCHROEDER. *Manifolds of Non-Positive Curvature.* Birkhäuser, Boston, 1985.

[Bow] BOWEN, R. Entropy and the fundamental group. In: *The Structure of Attractors in Dynamical Systems. (Proc. Conf., North Dakota State Univ., Fargo, N.D. (1977))*, 21–29. Lecture Notes in Math., 668, Springer, Berlin, 1978.

[Br-Ha] BRIDSON, M. R. and A. HAEFLIGER. *Metric Spaces of Non-Positive Curvature.* Grundlehren der mathematischen Wissenschaften, Vol. 319. Springer, Berlin, 1999.

[Din] DINABURG, E. I. A connection between various entropy characterizations of dynamical systems (Russian). *Izv. Akad. Nauk SSSR Ser. Mat. 35* (1971), 324–366.