Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 46 (2000)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE SATURATION CONJECTURE (AFTER A. KNUTSON AND T. TAO)

Autor: BUCH, Anders Skovsted / Fulton, William

Bibliographie

DOI: https://doi.org/10.5169/seals-64794

$\lambda^{(1)} \supset \lambda^{(2)} \supset \cdots \supset \lambda^{(n)} \supset \varnothing$, and from these partitions one successively fills in the entries in the northwest to southeast diagonal rows of the hive; the rhombus inequalities (1)–(3) are automatically satisfied.

To make the story complete, we recall why such contratableaux correspond to Littlewood-Richardson skew tableaux, using standard results about tableaux, as in [5]. However, it may be pointed out that these contratableaux are at least as easy to produce and enumerate as the more classical skew tableaux. First, the condition that $w(T) \cdot w(U(\mu))$ is a reverse lattice word, given that the number of times i occurs in T is $\nu_i - \mu_i$, is equivalent to asserting that $w(T) \cdot w(U(\mu))$ is Knuth equivalent to $w(U(\nu))$ [5, §5.2]. The rectification R of a contratableau T of shape λ is easily seen to be a tableau of shape λ, and with the same property that $w(R) \cdot w(U(\mu))$ is Knuth equivalent to $w(U(\nu))$. The correspondence between tableaux and contratableaux of shape λ is a bijection, by reversing the rectification process.

Now the condition that $w(R) \cdot w(U(\mu))$ be Knuth equivalent to $w(U(\nu))$ is equivalent to the condition that $R \cdot U(\mu) = U(\nu)$ in the plactic monoid of tableaux [5, §2.1]. It is easy to see, from the definition of multiplying tableaux by column bumping entries of the first tableau into the second [5, §A.2], that if R and S are tableaux with $R \cdot S = U(\beta)$, then S must be equal to $U(\alpha)$ for some partition α. This gives a correspondence between the set of tableaux R that we are looking at and the set of pairs (R, S) with R of shape λ, S of shape μ, whose product is the tableau $U(\nu)$. There is a standard construction [5, §5.1] between these pairs and the set of skew tableau on the shape ν/λ of content μ whose word is a reverse-lattice word.

REFERENCES

[1] BERENSTEIN, A. D. and A. V. ZELEVINSKY. Triple multiplicities for $\mathfrak{sl}(r+1)$ and the spectrum of the exterior algebra of the adjoint representation. *J. Algebraic Combin. 1* (1992), 7–22.

[2] BERTRAM, A., I. CIOCAN-FONTANINE and W. FULTON. Quantum multiplication of Schur polynomials. *J. Algebra 219* (1999), no. 2, 728–746.

[3] BUCH, A. S. and W. FULTON. Chern class formulas for quiver varieties. *Invent. Math. 135* (1999), no. 3, 665–687.

[4] CARRÉ, C. The rule of Littlewood-Richardson in a construction of Berenstein-Zelevinsky. *Internat. J. Algebra Comput. 1* (1991), 473–491.

[5] FULTON, W. Young tableaux. London Mathematical Society Student Texts, vol. 35. Cambridge University Press, 1997.

[6] —— Eigenvalues of sums of Hermitian matrices (after A. Klyachko). *Astérisque* (1998), no. 225, 225–269, Séminaire Bourbaki, Vol. 1997/98.

[7] GOODMAN, F. M. and H. WENZL. Littlewood-Richardson coefficients for Hecke algebras at roots of unity. *Adv. Math.* *82* (1990), 244–265.

[8] HORN, A. Eigenvalues of sums of Hermitian matrices. *Pacific J. Math.* *12* (1962), 225–241.

[9] KLYACHKO, A. A. Stable bundles, representation theory and Hermitian operators. *Selecta Math. (N.S.)* *4*, 1998, no. 3, 419–445.

[10] KNUTSON, A. and T. TAO. The honeycomb model of the Berenstein-Zelevinsky polytope I: Klyachko's saturation conjecture. *J. Amer. Math. Soc.* *12*, (1999), no. 4, 1055–1090.

[11] STURMFELS, B. and R. R. THOMAS. Variation of cost functions in integer programming. *Math. Programming* *77* (1997), no. 3, Ser. A, 357–387.

[12] ZELEVINSKY, A. Littlewood-Richardson semigroups. *MSRI Preprint*, 1997-044.

(Reçu le 5 janvier 1999)

Anders Skovsted Buch

Massachusetts Institute of Technology
Building 2, Room 248
77 Massachusetts Avenue
Cambridge, MA 02139
USA
e-mail: abuch@math.mit.edu