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Theorem 4 in the general case is also due to the first author. The present joint note grew from the second author's observation that the original proof can be simplified and be made more geometric by appealing to considerations in [5].

This work was completed during a visit by the second author to the Forschungsinstitut für Mathematik (FIM) of the ETH Zürich. He thanks the FIM for its hospitality and support.

## 2. STABLE ALMOST COMPLEX STRUCTURES

To derive Theorem 4(a) from Theorem 3(a) we merely have to show that the condition  $w_8(M) \in \text{Sq}^2 H^6(M; \mathbf{Z})$  is void in case I and equivalent to  $\chi(M) \equiv 0 \pmod{2}$  (i.e.  $w_8(M) = 0$ ) in case II.

Indeed, in case I we find an integral lift  $u$  of  $w_2(M)$  whose free part is indivisible. Then there is a dual element  $u' \in H^6(M; \mathbf{Z})$  such that  $uu' = 1 \in H^8(M; \mathbf{Z})$ . By the Wu formula it follows that

$$\text{Sq}^2 H^6(M; \mathbf{Z}) = w_2(M) H^6(M; \mathbf{Z}) = \rho_2(u H^6(M; \mathbf{Z})) = H^8(M; \mathbf{Z}_2).$$

In case II, on the other hand, we can lift  $w_2(M)$  to a torsion class  $u \in H^2(M; \mathbf{Z})$ , thus

$$\text{Sq}^2 H^6(M; \mathbf{Z}) = \rho_2(u H^6(M; \mathbf{Z})) = 0.$$

## 3. THE TOP-DIMENSIONAL OBSTRUCTION

Assume we have an almost complex structure  $J_0$  over  $M$  with a disc  $D^8$  removed (which is homotopy equivalent to the 7-skeleton of  $M$ ). Thinking of an almost complex structure  $J$  as a section of the  $\text{SO}_8/U_4$ -bundle associated to  $TM$ , we may interpret  $J_0$  as such a section defined only over the 7-skeleton of  $M$ . The obstruction  $\sigma(M, J_0)$  to extending  $J_0$  to an almost complex structure on  $M$  then lives in

$$H^8(M; \pi_7(\text{SO}_8/U_4)) \cong \pi_7(\text{SO}_8/U_4) \cong \mathbf{Z} \oplus \mathbf{Z}_2.$$

See [13] for references to the computations of the homotopy group above and others used below. The homotopy group involved here is in fact the