**Zeitschrift:** L'Enseignement Mathématique

**Herausgeber:** Commission Internationale de l'Enseignement Mathématique

**Band:** 46 (2000)

**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Kapitel:** TEACHING AND LEARNING WITH COMPUTER ALGEBRA SYSTEMS

## LANGUAGE ASPECTS OF ALGEBRA

This section considers theoretical and applied aspects of the languages and notations of algebra, in relation to teaching and learning. The evolution of algebra cannot be separated from the evolution of its language and notations. Historically the introduction of good notations has had enormous impact upon the development of algebra but a good notation for science may not be a good notation for learning. With new computer technology we are now seeing a flowering of new quasi-algebraic notations, which may offer, support or eventually enforce new notations. However, current theories of mathematics teaching and learning do not seem adequate to deal with learning about notation. It is therefore timely to focus on algebraic notations asking questions such as:

- How do theories of mathematics teaching and learning embrace the linguistic aspects of algebra and what can we propose to better take into account these aspects?
- Algebra is not a language but it has a language and the two cannot be dissociated. What does it mean to talk about algebra as a language and what are the implications of such a perspective?
- There is a wide range of theories of how mathematical concepts are learned and taught (in particular the constructivist theories) but learning a language is not just a matter of learning concepts. How do acknowledged theories of mathematics learning and teaching embrace the non-conceptual aspects of learning the language of algebra and what can we propose to better take these aspects into account?
- Would some changes of algebraic language contribute to the development of algebraic thinking, communication and understanding?
- Is it feasible and desirable to remove some of the ambiguities that are present in standard mathematical symbolism, for example in the use of the equals sign?
- Should some effort be made in the teaching of mathematics to explain and bridge differences in notation between algebra as it is taught in mathematics courses and algebra as it is used in other disciplines?
- What are the characteristics of good notation? What does mathematics education research have to say on this? Are some notational choices better for science but others better for learning?

## TEACHING AND LEARNING WITH COMPUTER ALGEBRA SYSTEMS

The advent of affordable computer systems and calculators that can perform symbolic calculations may lead to far-reaching changes in mathematics curricula and in mathematics teaching. This section addresses questions that arise from the increasing accessibility of computer symbolic manipulation. Answers to these questions will draw upon established research on the teaching and learning of algebra as well as reporting on recent experimental work. They may suggest new directions for research, including:

- For which students and when is it appropriate to introduce the use of a computer algebra system? When do the advantages of using such a system outweigh the effort that must be put into learning to use it? Are there activities using such systems that can be profitably undertaken by younger students?

- What algebraic insights and 'symbol sense' does the user of a computer algebra system need and what insights does the use of the systems bring?
- A strength of computer algebra systems is that they support multiple representations of mathematical concepts. How can this be used well? Might it be over-used?
- What are the relationships and interactions between different approaches and philosophies of mathematics teaching with the use of computer algebra systems?
- Students using different computational tools solve problems and think about concepts differently. Teachers have more options for how they teach. What impact does this have on teaching and learning? Which types of system support which kinds of learning? Can these differences be characterised theoretically?
- What should an algebra curriculum look like in a country where computer algebra systems are freely available? What 'by hand' skills should be retained?

## TECHNOLOGICAL ENVIRONMENTS

Recent research, curriculum development, and classroom practice have incorporated a number of technologies to help students develop meaning for various algebraic objects, ideas and processes. These include, but are not limited to, function graphers, spreadsheets, programming languages, one-line programming on calculators, and other specific computer software environments. [Here, we exclude computer algebra systems that are treated elsewhere.] In an attempt to characterise recent research and experience, this section will explore which aspects of specific computer/calculator environments are related to which kinds of algebra learning. This question will be explored in depth for specific examples of such technology, by addressing questions such as the following:

- For a given technological environment, what are the implicit assumptions regarding the underlying core aspects of algebra?
- Which important aspects of algebra are and are not touched upon by this environment?
- What kinds of algebra learning does this environment promote?
- What particular limitations are associated with the use of this environment and how can such limitations be dealt with?
- To what extent ought the goals of algebra education be affected by the availability of this technology?
- To which aspects of algebra learning does this particular technology make a distinctive, unique contribution?
- Are there documented long-term consequences of embedding this particular technology in an algebra curriculum, and if so, what are they?

Submissions for this section should include discussion of as many of the above sub-questions as possible, but with particular attention paid to the first two items above.