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On the other hand, if  $\zeta$  is any closed curve on  $S$  with a minimal number of intersections with  $\psi_i^j$  in its free homotopy class, then we can remove from  $S$  three points which do not lie on  $\zeta$  and such that two of these points lie on  $\psi_i^j$ . If we call the resulting surface  $S_\infty$  then  $\zeta$  defines a closed curve  $\zeta_\infty$  on  $S_\infty$ , and  $i(\zeta, \psi_i^j)$  equals the number of intersection points between  $\zeta_\infty$  and  $\gamma_i^j$  (where  $\gamma_i^j$  is given as before). This then shows that  $\mathcal{J}(\zeta_\infty, \gamma_i^j) \leq i(\zeta, \psi_i^j) = i([\zeta_\infty], \psi_i^j)$   $\square$

As an immediate consequence of Lemma 5.6 and Lemma 5.7 we obtain

**COROLLARY 5.8.** *The curves  $\psi_i^j$  on  $S$  are parametrizing for  $\mathcal{PL}$ . In particular, for every  $g \geq 2$  there is a family of  $6g + 3$  free homotopy classes on a closed surface of genus  $g$  which is parametrizing for  $\mathcal{PL}$ .*

**REMARK.** From [FLP] one immediately obtains a family of  $9g - 9$  closed curves on a closed surface of genus  $g$  which is parametrizing for  $\mathcal{PL}$ . To my knowledge, the minimal number of simple closed curves with this property is not known.

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