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HOLONOMY AND SUBMANIFOLD GEOMETRY

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ABSTRACT. We survey some applications of holonomic methods to the study of submanifold geometry, showing the consequences of some sort of extrinsic version of the de Rham Decomposition Theorem and of Berger's Theorem: the so-called *Normal Holonomy Theorem*. At the same time, from geometric methods in submanifold theory we sketch some very strong applications to the holonomy of Lorentzian manifolds. Moreover we give a conceptual modern proof of a result of Kostant for homogeneous spaces.

1. INTRODUCTION

A connection on a connected Riemannian manifold M can be interpreted as a way of comparing tangent spaces at different points, by means of parallel transport.

The parallel translation depends, in general, on the curve joining two points and this dependence is measured by the *holonomy group*, i.e. the linear group of isometries obtained by parallel transporting along based loops.

Actually holonomy groups can be defined for any connection on a vector bundle. For example, in this note we will be particularly interested in the holonomy group of the normal connection, called *normal holonomy group*.

Holonomy plays an important role in (intrinsic) Riemannian geometry, in the context of special Riemannian metrics, e.g., symmetric, Kähler, hyperkähler and quaternionic Kähler metrics.

The main purpose of this note is to survey the application of holonomic methods to the study of submanifold geometry and vice versa. Namely, we will sketch some very strong applications of geometric methods in submanifold

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