

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 48 (2002)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: HOLONOMY AND SUBMANIFOLD GEOMETRY
Autor: Console, Sergio / Di Scala, Antonio J. / Olmos, Carlos

Bibliographie
DOI: <https://doi.org/10.5169/seals-66066>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 02.04.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

one obtains a contradiction because totally geodesic submanifolds are obtained by intersecting the hyperbolic space H^n with Lorentzian subspaces. Thus, G must act transitively on each horosphere.

Finally, if G acts irreducibly then G must act transitively on the hyperbolic space and must be semisimple of noncompact type by a previous observation. Then, showing that the isotropy group at some point agrees with a maximal compact subgroup, the second part of the theorem follows from the theory of Riemannian symmetric spaces of noncompact type [He].

REFERENCES

- [AK] ALEKSEEVSKIĪ, D. V. and B. N. KIMEL'FELD. Structure of homogeneous Riemannian spaces with zero Ricci curvature (Russian). *Funkcional. Anal. i Priložen.*; English translation: *Funct. Anal. Appl.* 9 (1975), no. 2, 97–102.
- [BB] BÉRARD BERGERY, L. Sur la courbure des métriques riemanniennes invariantes des groupes de Lie et des espaces homogènes. *Ann. Sci. École Norm. Sup. (4)* 11 (1978), 543–576.
- [BI] BÉRARD BERGERY, L. and A. IKEMAKHEN. *On the Holonomy of Lorentzian Manifolds*. Proceedings of Symposia in Pure Mathematics, Volume 54 (1993), Part 2.
- [B1] BERGER, M. Sur les groupes d'holonomie des variétés à connexion affine et des variétés riemanniennes. *Bull. Soc. Math. France* 83 (1955), 279–330.
- [B2] ——— Les espaces symétriques non compacts. *Ann. Sci. École Norm. Sup.* 74 (1957), 85–177.
- [Br] BERNDT, J. Homogeneous hypersurfaces in hyperbolic spaces. *Math. Z.* 229 (1998), 589–600.
- [Be] BESSE, A. *Einstein Manifolds*. Springer, 1987.
- [CW] CARTER, S and A. WEST. Partial tubes about immersed manifolds. *Geom. Dedicata* 54 (1995), 145–169.
- [CO] CONSOLE, S. and C. OLMOS. Submanifolds of higher rank. *Quart. J. Math. Oxford Ser. (2)* 48 (1997), 309–321.
- [Da] DADOK, J. Polar coordinates induced by actions of compact Lie groups. *Trans. Amer. Math. Soc.* 288 (1985), 125–137.
- [D] DI SCALA, A. J. Minimal homogeneous submanifolds in Euclidean spaces. *Ann. Global Anal. Geom.* 21 (2002), 15–18.
- [D2] ——— Reducibility of complex submanifolds of the Euclidean space. *Math. Z.* 235 (2000), 251–257.
- [DO] DI SCALA, A. J. and C. OLMOS. The geometry of homogeneous submanifolds of hyperbolic space. *Math. Z.* 237 (2001), 199–209.
- [EO] ESCHENBURG, J.-H. and C. OLMOS. Rank and symmetry of Riemannian manifolds. *Comment. Math. Helv.* 69 (1994), 483–499.

- [FKM] FERUS, D., H. KARCHER and H.F. MÜNZNER. Cliffordalgebren und neue isoparametrische Hyperflächen. *Math. Z.* 177 (1981), 479–502.
- [He] HELGASON S. *Differential Geometry, Lie Groups and Symmetric Spaces*. Academic Press, New York, 1978.
- [H] HSIANG, W. Y. On the compact homogeneous minimal submanifolds. *Proc. Nat. Acad. Sci. U.S.A.* 56 (1966), 5–6.
- [HL] HSIANG, W. Y. and B. H. LAWSON. Minimal submanifolds of low cohomogeneity. *J. Differential Geom.* 5 (1971) 1–38.
- [HPTT] HEINTZE, E., R. PALAIS, C.-L. TERNG and G. THORBERGSSON. Hyperpolar actions and k -flat homogeneous spaces. *J. reine angew. Math.* 454 (1994), 163–179.
- [HO] HEINTZE, E. and C. OLMOS. Normal holonomy groups and s -representations. *Indiana Univ. Math. J.* 41 (3) (1992), 869–874.
- [HOT] HEINTZE, E., C. OLMOS and G. THORBERGSSON. Submanifolds with constant principal curvatures and normal holonomy groups. *Internat. J. Math.* 2 (1991), 167–175.
- [Iw] IWAHORI, N. Some remarks on tensor invariants of $O(n)$, $U(n)$, $Sp(n)$. *J. Math. Soc. Japan* 10 (1958), 145–160.
- [KN] S. KOBAYASHI and N. NOMIZU. *Foundations of Differential Geometry, Vol. I and II*. Interscience, New York, 1963, 1969.
- [K] KOSTANT, B. Holonomy and the Lie algebra of infinitesimal motions of a Riemannian manifold. *Trans. Amer. Math. Soc.* 80 (1955), 528–542.
- [N] NOMIZU, K. On infinitesimal holonomy and isotropy groups. *Nagoya Math. J.* 11 (1957), 111–114.
- [O1] OLMOS, C. The normal holonomy group. *Proc. Amer. Math. Soc.* 110 (1990), 313–818.
- [O2] ——— Isoparametric submanifolds and their homogeneous structures. *J. Differential Geom.* 38 (1993), 225–234.
- [O3] ——— Homogeneous submanifolds of higher rank and parallel mean curvature. *J. Differential Geom.* 39 (1994), 605–627.
- [O4] ——— Orbits of rank one and parallel mean curvature. *Trans. Amer. Math. Soc.* 347 (1995), 2927–2939.
- [OS] OLMOS, C. and C. SÁNCHEZ. A geometric characterization of the orbits of s -representation. *J. reine angew. Math.* 420 (1991), 195–202.
- [OSv] OLMOS, C. and M. SALVAI. Holonomy of homogeneous vector bundles and polar representations. *Indiana Univ. Math. J.* 44 (1995), 1007–1015.
- [OW] OLMOS, C. and A. WILL. Normal holonomy in Lorentzian space and submanifold geometry. *Indiana Univ. Math. J.* 50 (2002), 1777–1788.
- [OT1] OZEKI, H. and M. TAKEUCHI. On some types of isoparametric hypersurfaces in spheres I. *Tôhoku Math. J. (2)* 27 (1975), 515–559.
- [OT2] OZEKI, H. and M. TAKEUCHI. On some types of isoparametric hypersurfaces in spheres II. *Tôhoku Math. J. (2)* 28 (1976), 7–55.
- [PT1] PALAIS, R. and C.-L. TERNG. A general theory of canonical forms. *Trans. Amer. Math. Soc.* 300 (1987), 771–789.
- [PT2] PALAIS, R. and C.-L. TERNG. *Critical Point Theory and Submanifold Geometry*. Lect. Notes in Math. 1353. Springer, Berlin Heidelberg New York, 1988.

- [Sal] SALAMON, S. M. *Riemannian Geometry and Holonomy Groups*. Longman, Harlow, 1989.
- [Sim] SIMONS, J. On the transitivity of holonomy systems. *Ann. of Math. (2)* 76 (1962), 213–234.
- [St] STRÜBING, W. Isoparametric submanifolds. *Geom. Dedicata* 20 (1986), 367–387.
- [Sz] SZABÓ, Z. I. A short topological proof for the symmetry of 2-point homogeneous spaces. *Invent. Math.* 106 (1991), 61–64.
- [Te] TEZLAFF, K. Satz von Dadok und normale Holonomie. PhD thesis, Univ. Augsburg (1993).
- [Th] THORBERGSSON, G. Isoparametric foliations and their buildings. *Ann. of Math. (2)* 133 (1991), 429–446.
- [Wa] WANG, H.-C. Two-point homogeneous spaces. *Ann. of Math. (2)* 55 (1952), 177–191.
- [W] WILL, A. Homogeneous submanifolds of the hyperbolic space. *Rend. Sem. Mat. Univ. Politec. Torino* 56 (1998), 1–4.

(Reçu le 14 mai 2001)

S. Console

Dipartimento di Matematica
Università di Torino
Via Carlo Alberto 10
I-10123 Torino
Italy
e-mail: console@dm.unito.it

A. J. Di Scala

C. Olmos

Fa. M. A. F.
Universidad Nacional de Córdoba
Ciudad Universitaria
5000 Córdoba
Argentina
e-mail: discala@mate.uncor.edu
olmos@mate.uncor.edu