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A TRIPLE RATIO ON THE UNITARY STIEFEL MANIFOLD

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ABSTRACT. For the unitary Stiefel manifold S realized as the Shilov boundary of the unit ball D in $\mathrm{Mat}(p \times q, \mathbf{C})$, we construct characteristic invariants for the (generic) orbits of the conformal group $\mathbf{PSU}(p,q)$ in $S \times S \times S$. The construction uses the automorphy kernel of the bounded symmetric domain.

INTRODUCTION

Let $D = G/K$ be a bounded symmetric domain in a complex vector space $\mathbf{C}^N$, and let S be its Shilov boundary. The action of G extends to S and this action is transitive on S. It is generally referred to in the literature as the *conformal action* of G on S. One can show that the action is almost 2-transitive in the sense that G has a dense open orbit in $S \times S$. Hence it is a natural question to look for the G-orbits in $S \times S \times S$ and for characteristic invariants of this action. If D happens to be of tube type (in which case $\dim_{\mathbf{R}} S = \dim_{\mathbf{C}} D$), this question was solved in [CØ]. There are a finite number of open orbits in $S \times S \times S$, and the (generalized) *Maslov index* we constructed is a characteristic invariant for the G-action. In the case of the unit ball in $\mathbf{C}^2$, the Shilov boundary coincides with the topological boundary, namely the unit sphere $S = \mathbf{S}^3$. In [Ca], E. Cartan constructed a (real-valued) invariant for triples on S (he called S the "hypersphere"). Independently (and more than 50 years later) Korányi and Reimann studied the case of the unit ball in $\mathbf{C}^n$ (see [KR]). Through the Cayley transform, the problem is changed into an equivalent problem for the Heisenberg group $\mathbf{H}_n$ under the action of its conformal group $G = \mathbf{PSU}(n+1,1)$. For this situation, they studied a complex cross ratio on $\mathbf{H}_n$, from which they were able (in a rather indirect way) to construct a (real-valued) invariant for triples, which characterizes the G-orbits of triples in $\mathbf{H}_n$. Here we solve the problem for the case where D