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## A TRIPLE RATIO ON THE UNITARY STIEFEL MANIFOLD

by Jean-Louis CLERC

ABSTRACT. For the unitary Stiefel manifold  $S$  realized as the Shilov boundary of the unit ball  $D$  in  $\text{Mat}(p \times q, \mathbf{C})$ , we construct characteristic invariants for the (generic) orbits of the conformal group  $\mathbf{PSU}(p, q)$  in  $S \times S \times S$ . The construction uses the automorphy kernel of the bounded symmetric domain.

### INTRODUCTION

Let  $D = G/K$  be a bounded symmetric domain in a complex vector space  $\mathbf{C}^N$ , and let  $S$  be its Shilov boundary. The action of  $G$  extends to  $S$  and this action is transitive on  $S$ . It is generally referred to in the literature as the *conformal action* of  $G$  on  $S$ . One can show that the action is almost 2-transitive in the sense that  $G$  has a dense open orbit in  $S \times S$ . Hence it is a natural question to look for the  $G$ -orbits in  $S \times S \times S$  and for characteristic invariants of this action. If  $D$  happens to be of tube type (in which case  $\dim_{\mathbf{R}} S = \dim_{\mathbf{C}} D$ ), this question was solved in [CØ]. There are a finite number of open orbits in  $S \times S \times S$ , and the (generalized) *Maslov index* we constructed is a characteristic invariant for the  $G$ -action. In the case of the unit ball in  $\mathbf{C}^2$ , the Shilov boundary coincides with the topological boundary, namely the unit sphere  $S = \mathbf{S}^3$ . In [Ca], E. Cartan constructed a (real-valued) invariant for triples on  $S$  (he called  $S$  the “hypersphere”). Independently (and more than 50 years later) Korányi and Reimann studied the case of the unit ball in  $\mathbf{C}^n$  (see [KR]). Through the Cayley transform, the problem is changed into an equivalent problem for the Heisenberg group  $\mathbf{H}_n$  under the action of its conformal group  $G = \mathbf{PSU}(n + 1, 1)$ . For this situation, they studied a complex cross ratio on  $\mathbf{H}_n$ , from which they were able (in a rather indirect way) to construct a (real-valued) invariant for triples, which characterizes the  $G$ -orbits of triples in  $\mathbf{H}_n$ . Here we solve the problem for the case where  $D$