

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 49 (2003)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON THE ENTROPY OF HOLOMORPHIC MAPS
Autor: GROMOV, Mikhaïl
Kapitel: §6. Mean curvature
DOI: <https://doi.org/10.5169/seals-66687>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 02.04.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

§6. MEAN CURVATURE

Let X be a closed n -dimensional manifold with a Riemannian metric g . Suppose that iterated graphs $\Gamma_k \subset X^k$ are smooth of dimension n . Denote by $Cu(\gamma)$, $\gamma \in \Gamma_k$, the absolute value of the mean curvature of Γ_k at γ . Set

$$\text{lome}_g \Gamma = \limsup_{k \rightarrow \infty} \frac{1}{k} \log \left(1 + \int_{\Gamma_k} [Cu(\gamma)]^n d\gamma \right).$$

When Γ_k are minimal and $\text{lome}_g = 0$ we know that $h \leq$ "lov".

More generally,

$$(6.0) \quad h(\gamma) \leq \text{lov } \Gamma + \text{lome}_g \Gamma.$$

Proof. Despite the possible dependence of "lome" upon the choice of g , we can proceed as before and reduce (6.0) to the following local estimate:

Take V in the Euclidean space $\mathbf{R}^{\ell=kn}$ and suppose its boundary does not intersect the ball $B_{2\epsilon}$ centered at $v_0 \in V$. Then

$$(6.1) \quad \epsilon^{-n} \text{Vol } V + \int_V Cu^n(v) dv \geq C_1 \ell^{C_2},$$

where C_1 and C_2 are constants depending only on n .

To prove (6.1) we consider the normal bundle N of V and its canonical map F into \mathbf{R}^ℓ . The Jacobian J of F at a point $v + \nu t$ (where $v \in V$, and ν is the unit vector at v normal to V) is equal to $\prod_{i=1}^n (1 + k_i t)$, where k_i are the principal curvatures in the direction ν .

If the distance from $v + \nu t$ to V is equal to t , then $1 + k_i t \geq 0$, $i = 1, \dots, n$, and so

$$(6.2) \quad J \leq A_n (1 + t^n Cu^n(v)).$$

Now we observe that every point of the ball B_ϵ can be joined by a shortest normal with V and so

$$C_\ell \epsilon^\ell = \text{Vol } B_\epsilon \leq A_n C_{\ell-n} \epsilon^{\ell-n} \int_V (1 + \epsilon^n Cu^n(v)) dv,$$

where C_ℓ and $C_{\ell-n}$ are volumes of unit balls in \mathbf{R}^ℓ and $\mathbf{R}^{\ell-n}$. The last inequality implies (6.1) and so (6.0) is proved.

The inequality (6.2) was extended by Karcher and Heinze to general Riemannian manifolds. Discussions with Karcher about such inequalities influenced my reasoning in this section.