(4) BLOWING UP

Let us take $W \subset V_0$ and an endomorphism $f: V_0 \to V_0$ such that $f^{-1}(W) = W$. The endomorphism f can be sometimes lifted to the manifold V obtained by blowing up W.

EXAMPLE. $V_0 = \mathbf{C}P^1 \times \mathbf{C}P^1$, W is the single point $(0,0)$, and $f: (z_1, z_2) \mapsto (z_1^p, z_2^p)$.

(5) CONCLUDING REMARKS

A typical compact complex manifold has very few endomorphisms. For example, manifolds with nontrivial Kobayashi volume have no endomorphisms of degree ≥ 2. Do Grassmann manifolds have such endomorphisms? (No, see [3′].)

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