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Autor: Meinrenken, Eckhard

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THE BASIC GERBE OVER A COMPACT SIMPLE LIE GROUP

by Eckhard MEINRENKEN

ABSTRACT. Let G be a compact, simply connected simple Lie group. We give a construction of an equivariant gerbe with connection on G, with equivariant 3-curvature representing a generator of $H^3_G(G,\mathbf{Z})$. Among the technical tools developed in this context is a gluing construction for equivariant bundle gerbes.

1. INTRODUCTION

Let G be a compact, simply connected simple Lie group, acting on itself by conjugation. It is well-known that the cohomology of G, and also its equivariant cohomology, is trivial in degree less than three and that $H^3(G,\mathbf{Z})$ and $H^3_G(G,\mathbf{Z})$ are canonically isomorphic to $\mathbf{Z}$. The generator of $H^3(G,\mathbf{Z})$ is represented by a unique bi-invariant differential form $\eta \in \Omega^3(G)$, admitting an equivariantly closed extension $\eta_G \in \Omega^3_G(G)$ in the complex of equivariant differential forms. Our goal in this paper is to give an explicit, finite-dimensional description of an equivariant gerbe over G, with equivariant 3-curvature η_G.

A number of constructions of gerbes over compact Lie groups can be found in the literature, using different models of gerbes and valid in various degrees of generality. The differential geometry of gerbes was initiated by Brylinski's book [8], building on earlier work of Giraud. In this framework gerbes are viewed as sheafs of groupoids satisfying certain axioms. Brylinski gives a general construction of a gerbe with connection, for any integral closed 3-form on any 2-connected manifold M. The argument uses the path fibration $P_0M \to M$, and is similar to the well-known construction of a line bundle with connection out of a given integral closed 2-form on a simply connected manifold. In a later paper [9], Brylinski gives a finite-dimensional description of the sheaf of groupoids defining the basic gerbe for any compact Lie group G.