

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 49 (2003)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE BASIC GERBE OVER A COMPACT SIMPLE LIE GROUP
Autor: Meinrenken, Eckhard
Anhang: Appendix A. Proof of Lemma 4.4
DOI: <https://doi.org/10.5169/seals-66691>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 02.04.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

REMARK 6.2. Z. Shahbazi has proved that if \mathcal{G} is a gerbe with connection over a manifold M , with curvature 3-form η , and $\Phi: N \rightarrow M$ is a map with $\Phi^*\eta + d\omega = 0$, then the pull-back gerbe $\Phi^*\mathcal{G}$ admits a pseudo-line bundle, with ω as its error 2-form, if and only if the pair (η, ω) defines an integral element of the relative de Rham cohomology $H^3(\Phi, \mathbf{R})$. This means that for any smooth 2-cycle $S \subset N$, and any smooth 3-chain $B \subset M$ with boundary $\Phi(S)$, one must have $\int_B \eta - \int_S \omega \in \mathbf{Z}$. The particular case where the target of Φ is a Lie group G is relevant for the pre-quantization of group-valued moment maps [1].

APPENDIX A. PROOF OF LEMMA 4.4

In this Appendix we prove Lemma 4.4, concerning the construction of a certain cover U_I of M from a given cover V_j . Write $M = \coprod_I A_I$ where

$$A_I = \bigcap_{i \in I} V_i \setminus \bigcup_{j \notin I} V_j.$$

Notice that $\bar{A}_I \subset \bigcup_{J \subset I} A_J$. By induction on the cardinality $k = |I|$ we will construct open sets $U_I \subset V_I$, having the following properties:

- (a) the closure \bar{U}_I does not meet \bar{U}_J for $|J| \leq |I|$ unless $J \subset I$,
- (b) each \bar{A}_I is contained in the union of U_J with $J \subset I$.

The induction starts at $k = 0$, taking $U_\emptyset = \emptyset$. Suppose we have constructed open sets U_I with $\bar{U}_I \subset V_I$ for $|I| < k$, such that the properties (a), (b) hold for all $|I| < k$. For $|I| = k$ consider the subsets

$$B_I := A_I \setminus \left(\bigcup_{J \subset I, |J| < k} U_J \right).$$

Note that (unlike A_I) the set B_I is closed. B_I does not meet \bar{A}_J unless $I \subset J$, and it also does not meet \bar{U}_J for $|J| < k$ unless $J \subset I$. That is, B_I is disjoint from

$$C_I := \bigcup_{J \not\subset I, |J| < k} \bar{U}_J \cup \bigcup_{K \not\subset I} \bar{A}_K.$$

Choose open sets U_I for $|I| = k$ with $B_I \subset U_I \subset \bar{U}_I \subset M \setminus C_I$, and such that the closures of the sets U_I for distinct I with $|I| = k$ are disjoint. The new collection of subsets will satisfy the properties (a), (b) for $|I| \leq k$. We next show that $V'_i = M \setminus \bigcup_{J \not\supset i} \bar{U}_J$ is a cover of M . Write $M = \coprod_I D_I$ with $D_I = \bar{U}_I \setminus \bigcup_{|J| < |I|} \bar{U}_J$. Then $D_I \cap \bar{U}_J = \emptyset$ unless $I \subset J$, so D_I is contained

in each V'_i with $i \in I$. In particular $\bigcup_i V'_i = M$. Finally $\overline{V'_i} \subset \bigcup_{I \ni i} \overline{U}_I \subset V_i$. This completes the proof of Lemma 4.4. Note that if the V_i were invariant under an action of a compact group G , the U_I could be taken G -invariant also.

REFERENCES

- [1] ALEKSEEV, A., A. MALKIN and E. MEINRENKEN. Lie group valued moment maps. *J. Differential Geom.* 48 (1998), 445–495.
- [2] BEHREND, K., P. XU and B. ZHANG. Equivariant gerbes over compact simple Lie groups. Preprint, 2002.
- [3] BERLINE, N., E. GETZLER and M. VERGNE. *Heat Kernels and Dirac Operators*. Grundlehren der mathematischen Wissenschaften 298. Springer-Verlag, Berlin-Heidelberg-New York, 1992.
- [4] BOTT, R. and L. TU. *Differential Forms in Algebraic Topology*. Graduate Texts in Mathematics 82. Springer-Verlag, New York, 1982.
- [5] BOURBAKI, N. *Éléments de mathématique. Groupes et algèbres de Lie*. Chapitre IV–VI. Hermann, Paris, 1968.
- [6] BRÖCKER, T. and T. TOM DIECK. *Representations of Compact Lie Groups*. Graduate Texts in Mathematics 98. Springer-Verlag, Berlin-Heidelberg-New York, 1985.
- [7] BRYLINSKI, J.-L. Gerbes on complex reductive Lie groups. arXiv:math.DG/0002158.
- [8] ———. *Loop Spaces, Characteristic Classes and Geometric Quantization*. Birkhäuser, Boston, 1993.
- [9] BRYLINSKI, J.-L. and D. A. MCLAUGHLIN. The geometry of degree-four characteristic classes and of line bundles on loop spaces. I. *Duke Math. J.* 75 (1994), 603–638.
- [10] CHATTERJEE, D. On the construction of Abelian gerbe. Ph.D. thesis, University of Cambridge, 1998.
- [11] DUISTERMAAT, J.J. and J.A.C. KOLK. *Lie Groups*. Springer-Verlag, Berlin, 2000.
- [12] DUPONT, J.L. Simplicial de Rham cohomology and characteristic classes of flat bundles. *Topology* 15 (1976), 233–245.
- [13] GAWĘDZKI, K. and N. REIS. WZW branes and gerbes. arXiv:hep-th/0205233.
- [14] GOMI, K. Connections and curvings on lifting bundle gerbes. arXiv:math.DG/0107175.
- [15] GUILLEMIN, V. and S. STERNBERG. *Supersymmetry and Equivariant de Rham Theory*. Springer-Verlag, 1999.
- [16] GURUPRASAD, K., J. HUEBSCHMANN, L. JEFFREY and A. WEINSTEIN. Group systems, groupoids, and moduli spaces of parabolic bundles. *Duke Math. J.* 89 (1997), 377–412.