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connected component by $SO(3)$ (i.e. by “any” group of the same rank), or the group of components by $\mathbf{Z}/4$ (i.e. by “any” group of the same size), will force the extension to be split.

Proof. The assertions about the connected component and the group of components are clear. Let us show that the extension associated to G is not split. Let us denote $[g, \gamma] \in G$ the image of $(g, \gamma) \in SU(2) \times D_8$ under the canonical projection. Let \mathbf{S}^1 denote the standard maximal torus in $SU(2)$, and let N denote its normalizer in G . We have

$$N = \{[t, e] : t \in \mathbf{S}^1\} \amalg \{[jt, e] : t \in \mathbf{S}^1\} \amalg \{[t, r] : t \in \mathbf{S}^1\} \amalg \{[jt, r] : t \in \mathbf{S}^1\} \\ \amalg \{[t, s] : t \in \mathbf{S}^1\} \amalg \{[jt, s] : t \in \mathbf{S}^1\} \amalg \{[t, rs] : t \in \mathbf{S}^1\} \amalg \{[jt, rs] : t \in \mathbf{S}^1\}.$$

By contradiction, suppose that the extension associated to G is split, i.e. there exists a section. As $\mathbf{Z}/2 \times \mathbf{Z}/2$ is abelian, thus nilpotent, we deduce, by Proposition 5.4, that the extension associated to N is also split. We want to show that this is not possible by considering the elements of order 2 in N . For $n = 0, 1$, a straightforward calculation shows that in the component corresponding to $r^n s$, an element $[t, r^n s]$ is of order 2 if and only if $t = \pm \mathbf{1}$, and that the sub-component $\{[jt, r^n s] : t \in \mathbf{S}^1\}$ does not contain any element of order 2. Two of the three non-trivial elements in $\Gamma \cong \mathbf{Z}/2 \times \mathbf{Z}/2$ must thus be mapped by the section to $[\pm \mathbf{1}, s]$ and $[\pm \mathbf{1}, rs]$. Therefore, as the section is a homomorphism, the image of the third non-trivial element is

$$[\pm \mathbf{1}, rs] \cdot [\pm \mathbf{1}, s] = [\pm \mathbf{1}, r],$$

which is not of order 2. A contradiction that shows that the extension associated to G is not split.

The property of minimality follows by Theorem 5.3, and by the fact that any extension with $SO(3)$ as normal subgroup is a direct product (because $SO(3)$ is complete, i.e. centerless and with trivial outer automorphism group). \square

REFERENCES

- [1] ADAMS, J.F. Maps between classifying spaces, II. *Invent. Math.* 49 (1978), 1–65.
- [2] ADEM, A. and R.J. MILGRAM. *Cohomology of Finite Groups*. Springer, 1994.
- [3] BOREL, A. and J. DE SIEBENTHAL Les sous-groupes fermés de rang maximum des groupes de Lie clos. *Comment. Math. Helv.* 23 (1949), 200–221.
- [4] BOURBAKI, N. *Groupes et algèbres de Lie (Chapitres 2 et 3)*. Hermann, 1972.
- [5] ——— *Groupes et algèbres de Lie (Chapitre 9)*. Masson, 1983.

- [6] BROWN, K. S. *Cohomology of Groups*. GTM 87. Springer, 1982.
- [7] CURTIS, M., A. WIEDERHOLD and B. WILLIAMS. Normalizers of maximal tori. In: *Lecture Notes in Math.* 418, 31–47. Springer, 1974.
- [8] DE SIEBENTHAL, J. Sur les sous-groupes fermés d'un groupe de Lie clos. *Comment. Math. Helv.* 25 (1951), 210–256.
- [9] — Sur les groupes de Lie compacts non connexes. *Comment. Math. Helv.* 31 (1956/57), 41–89.
- [10] DYNKIN, E. B. Automorphisms of semi-simple Lie algebras. *Doklady Akad. Nauk SSSR (N.S.)* 76 (1951), 629–632.
- [11] EILENBERG, S. and S. MAC LANE. Cohomology theory in abstract groups, II. Group extensions with non-abelian kernel. *Ann. of Math. (2)* 48 (1947), 326–341.
- [12] HÄMMERLI, J.-F. Une généralisation de la notion de tore maximal dans les groupes de Lie compacts non connexes. Travail de diplôme. Université de Neuchâtel, 1995.
- [13] — Normalizers of maximal tori and classifying spaces of compact Lie groups. Ph.D. thesis. University of Neuchâtel, 2000.
- [14] HOFMANN, K. H. and S. A. MORRIS. *The Structure of Compact Groups*. de Gruyter, 1998.
- [15] KIRILLOV, A. A. *Elements of the Theory of Representations*. Springer, 1976.
- [16] MAC LANE, S. *Homology*. Springer, 1963.
- [17] MATTHEY, M. Sur les sous-groupes principaux et les normalisateurs de tores maximaux dans les groupes de Lie compacts connexes. Travail de diplôme. Université de Neuchâtel, 1995.
- [18] MCINNES, B. Disconnected forms and the standard group. *J. Math. Phys. (8)* 38 (1997), 4354–4362.
- [19] OLIVER, B. The representation ring of a compact Lie group revisited. *Comment. Math. Helv.* 73 (1998), 353–378.
- [20] OSSE, A. λ -structures and representation rings of compact connected Lie groups. *J. Pure Appl. Algebra* 121 (1997), 69–93.
- [21] ROBINSON, D. J. S. *A Course in the Theory of Groups*. GTM 80 (2nd ed.). Springer, 1996.
- [22] SEGAL, G. The representation ring of a compact Lie group. *Inst. Hautes Études Sci. Publ. Math.* 34 (1968), 113–128.
- [23] TOM DIECK, T. *Transformation Groups and Representation Theory*. Lecture Notes in Math. 766. Springer, 1979.

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