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ATIYAH'S L^2 -INDEX THEOREM

by Indira CHATTERJI and Guido MISLIN

1. INTRODUCTION

The L^2 -Index Theorem of Atiyah [1] expresses the index of an elliptic operator on a closed manifold M in terms of the G -equivariant index of some regular covering \tilde{M} of M , with G the group of covering transformations. Atiyah's proof is analytic in nature. Our proof is algebraic and involves an embedding of a given group into an acyclic one, together with naturality properties of the indices.

2. REVIEW OF THE L^2 -INDEX THEOREM

The main reference for this section is Atiyah's paper [1]. All manifolds considered are smooth Riemannian, without boundary. Covering spaces of manifolds carry the induced smooth and Riemannian structure. Let M be a closed manifold and let E, F denote two complex (Hermitian) vector bundles over M . Consider an elliptic pseudo-differential operator

$$D: C^\infty(M, E) \rightarrow C^\infty(M, F)$$

acting on the smooth sections of the vector bundles. One defines its space of solutions

$$S_D = \{s \in C^\infty(M, E) \mid Ds = 0\} .$$

The complex vector space S_D has finite dimension (see [13]), and so has S_{D^*} the space of solutions of the adjoint D^* of D where

$$D^*: C^\infty(M, F) \rightarrow C^\infty(M, E)$$