**Zeitschrift:** L'Enseignement Mathématique
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique
**Band:** 50 (2004)
**Heft:** 1-2: L'enseignement mathématique

**Artikel:** Linear functional equations and Shapiro's conjecture
**Autor:** Laczkovich, M.
**Bibliographie**
**DOI:** https://doi.org/10.5169/seals-2642

**Nutzungsbedingungen**
Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Siehe Rechtliche Hinweise.

**Conditions d'utilisation**
L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. Voir Informations légales.

**Terms of use**
The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. See Legal notice.

**Download PDF:** 09.03.2025

**ETH-Bibliothek Zürich, E-Periodica, https://www.e-periodica.ch**

This proves that $\mathrm{sp}(L)$ is finite. If $e^{\lambda_1 z}, \ldots, e^{\lambda_s z}$ are the only exponential functions contained in $L$, then every exponential polynomial contained in $L$ must be of the form $\sum_{i=1}^{s} p_i(z)\, e^{\lambda_i z}$, where $p_1, \ldots, p_s$ are polynomials. Since the set of all polynomials is dense in $C(\mathbf{R})$ and $L \neq C(\mathbf{R})$, it follows that the degrees of $p_1, \ldots, p_s$ must be bounded. As the set of exponential polynomials is dense in $L$, we find that each element of $L$ is an exponential polynomial, which completes the proof of Theorem 2 in the case when $h \equiv 0$.

The general case can be reduced to the previous one in the same way as in the proof of Theorem 1. Again, it is enough to show that $f_n$ is an exponential polynomial. Since $\Delta_b f_n$ satisfies the homogeneous version of (1), it follows that $\Delta_b f_n$ is an exponential polynomial for every $b$. Therefore, by Carroll's theorem [2], $f_n$ is also an exponential polynomial. □

## REFERENCES


[1] BAKER, J. A. Functional equations, distributions and approximate identities. *Canad. J. Math. 42* (1990), 696–708.

[2] CARROLL, F. W. A difference property for polynomials and exponential polynomials on Abelian locally compact groups. *Trans. Amer. Math. Soc. 114* (1965), 147–155.

[3] EVEREST, G. R. and A. J. VAN DEN POORTEN. Factorization in the ring of exponential polynomials. *Proc. Amer. Math. Soc. 125* (1997), 1293–1298.

[4] JÁRAI, A. On Lipschitz property of solutions of functional equations. *Aequationes Math. 47* (1994), 69–78.

[5] KAHANE, J.-P. Sur quelques problèmes d'unicité et de prolongement, relatifs aux fonctions approchables par des sommes d'exponentielles. *Ann. Inst. Fourier (Grenoble) 5* (1953–54), 39–130.

[6] —— *Lectures on Mean Periodic Functions*. Tata Institute, 1956.

[7] KELETI, T. Difference functions of periodic measurable functions. *Fund. Math. 157* (1998), 15–32.

[8] VAN DEN POORTEN, A. J. and R. TIJDEMAN. On common zeros of exponential polynomials. *L'Enseignement Math. (2) 21* (1975), 57–67.

[9] RITT, J. F. A factorization theory for functions $\sum_{i=1}^{n} a_i e^{\alpha_i x}$. *Trans. Amer. Math. Soc. 29* (1927), 584–596.

[10] SCHWARTZ, L. Théorie générale des fonctions moyenne-périodiques. *Ann. of Math. (2) 48* (1947), 857–929.

[11] SHAPIRO, H. S. The expansion of mean-periodic functions in series of exponentials. *Comm. Pure Appl. Math. 11* (1958), 1–21.

[12] ŚWIATAK, H. On the regularity of the locally integrable solutions of the functional equations $\sum_{i=1}^{k} a_i(x,t)f(x+\phi_i(t)) = b(x,t)$. *Aequationes Math. 1* (1968), 6–19.

*(Reçu le 27 juillet 2003)*

M. Laczkovich

Department of Analysis
Eötvös Loránd University
Pázmány Péter sétány 1/C
H-Budapest
Hungary 1117
*e-mail:* laczk@cs.elte.hu

and

Department of Mathematics
University College London
Gower Street
GB-London WC1E 6BT
England