Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 50 (2004)
Heft: 1-2: L'enseignement mathématique

Artikel: Lattices, l2-Betti numbers, deficiency, and knot groups

Autor: Eckmann, Beno

Bibliographie

DOI: https://doi.org/10.5169/seals-2643

Since the torus knot groups are the only knot groups with non-trivial center the groups of all other knots cannot be lattices in $\mathbf{R} \times PSL_2(\mathbf{R})$.

9.2. So we now consider knots which are not torus knots; their groups can be lattices in $PSL_2(\mathbf{C})$ only. As for $PSL_2(\mathbf{C})$, it is the isometry group of hyperbolic 3-space $\mathbf{H}^3$. The knot complement C is a Haken 3-manifold with zero-Euler characteristic. It is atoroidal precisely if the respective knot does not have a companion; we use here "companion" in the sense of non-trivial companion. Indeed the boundary torus of a regular neighborhood of a companion would be a non-boundary-parallel incompressible surface in C. Thus by Thurston's Hyperbolization Theorem the interior of the complement of a knot without companion can be given a hyperbolic structure with finite volume coming from $\mathbf{H}^3/G$. In other words that knot group G is a lattice in $PSL_2(\mathbf{C})$ and the interior of C can be identified with the open manifold $\mathbf{H}^3/G$.

Concerning the notion of companion see [Ro, p. 111]. For concepts related to the Hyperbolization Theorem and to properties of 3-manifolds we refer to [K].

9.3. If the knot has a companion then the knot complement is not atoroidal and its interior does not admit a hyperbolic structure [K, Cor.4.63]. It follows that G cannot be a lattice in (the only remaining possibility) $PSL_2(\mathbf{C})$. We sketch the proof: If G is such a lattice then $\mathbf{H}^3/G$ is a $K(G,1)$-manifold as well, thus homotopy equivalent to C. The homotopy equivalence can be turned into a diffeomorphism mapping $\mathbf{H}^3/G$ to the interior of the knot complement C which thus would receive a hyperbolic structure with finite volume.

THEOREM 9.1. *Torus knot groups are lattices in* $\mathbf{R} \times PSL_2(\mathbf{R})$. *As for other groups of knots, those of knots without companion are lattices in* $PSL_2(\mathbf{C})$, *and those of knots with companion are not lattices in any connected Lie group.*

REFERENCES

[A] AUSLANDER, L. On radicals of discrete subgroups of Lie groups. *Amer. J. Math. 85* (1963), 145–150.

[B] BROWN, K. S. *Cohomology of Groups*. Springer-Verlag, 1982.

[B-S] BOREL, A. and J-P. SERRE. Corners and arithmetic groups. *Comment. Math. Helv. 48* (1974), 436–491.

[B-W] BOREL, A. and N. WALLACH. Continuous cohomology, discrete subgroups, and representations of reductive groups. *Math. Surveys and Monographs 67.* Amer. Math. Soc., 2000.

[Ch-G] CHEEGER, J. and M. GROMOV. L^2-cohomology and group cohomology. *Topology 25* (1986), 189–215.

[Ch-G2] CHEEGER, J. and M. GROMOV. Bounds on the von Neumann dimension of L^2-cohomology and the Gauss-Bonnet theorem for open manifolds. *J. Differential Geom. 21* (1985), 1–34.

[C] COHEN, D. E. *Groups of Cohomological Dimension One.* LNM *245*. Springer-Verlag 1972.

[C-M] CONNES, A. and H. MOSCOVICI. The L^2-index theorem for homogeneous spaces of Lie groups. *Ann. of Math. (2) 115* (1982), 291–330.

[D] DODZIUK, J. L^2-harmonic forms on rotationally symmetric Riemannian manifolds *Proc. Amer. Math. Soc. 77* (1979), 395–400.

[D2] —— De Rham-Hodge theory for L^2-cohomology of infinite coverings. *Topology 16* (1977), 157–165.

[E] ECKMANN, B. Introduction to ℓ_2-methods in topology. *Israel J. Math. 117* (2000), 183–219.

[E2] —— 4-manifolds, group invariants, and ℓ_2-Betti numbers. *L'Enseign. Math. 43* (1997), 271–279.

[G] GABORIAU, D. Invariants ℓ_2 des relations d'équivalence et des groupes. *Inst. Hautes Études Sci. Publ. Math. 95* (2002), 93–150.

[H] HILLMAN, J. On L^2-homology and asphericity. *Israel J. Math. 99* (1997), 271–283.

[K] KAPOVICH, M. *Hyperbolic Manifolds and Discrete Groups.* Progress in Mathematics *183*. Birkhäuser, 2000.

[L] LOTT, J. Deficiencies of lattice subgroups of Lie groups. *Bull. London Math. Soc. 31* (1999), 191–195.

[L-L] LOTT, J. and W. LÜCK. L^2-topological invariants of 3-manifolds. *Invent. Math. 120* (1995), 15–60.

[Lü] LÜCK, W. *L^2-Invariants: Theory and Applications to Geometry and K-Theory.* Springer-Verlag, 2003.

[S] SERRE, J-P. *Cohomologie des groupes discrets.* Prospects in Mathematics, Ann. of Math. Studies *70*. Princeton, 1971.

[R] RAGUNATHAN, M. S. *Discrete Subgroups of Lie Groups.* Ergebnisse *68*. Springer-Verlag, 1972.

[Ro] ROLFSEN, D. *Knots and Links*, 2nd printing. Publish or Perish, 1999.

(Reçu le 3 novembre 2003)

Beno Eckmann

Forschungsinstitut für Mathematik
ETH Zentrum
Rämistrasse 101
CH-8092 Zurich
Switzerland

Leere Seite
Blank page
Page vide