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STEP 6. Note that $\bar{A}$ is also the normalization of B. We will compute $\dim(\bar{A}/B)$.

By [Se], p. 59, Formula (3), the question can again be reduced to the complete case. Let D be the completion of B. Then

$$D = k[[w,u]]/\Big(\epsilon u^{m-n} - w^n \prod_{1\le i\le m}(1+\lambda_i u)\Big).$$

Find $\alpha \in D$ such that $\alpha^{m-n} = \epsilon^{-1}\prod_{1\le i\le m}(1+\lambda_i u)$. Define $U = u/\alpha$. Then $D \simeq k[[w,U]]/(U^{m-n} - w^n)$. Now we can apply Lemma 4 to get $\dim(\bar{D}/D) = \{(n-1)(m-n-1) - 1 + d\}/2$.

STEP 7. Finally we find that $\delta_P = \dim(\bar{A}/A) = \dim(B/A) + \dim(\bar{A}/B) = \{(m-1)(m-n-1) - 1 + d\}/2$. Thus

$$(N-1)(N-2)/2 - \delta_P = \{(m-1)(n-1) + 1 - d\}/2$$

because $N = \max\{m,n\} = m$. This completes the proof of Theorem 2.

REMARKS.

(1) From the above proof, it is clear that Theorem 2 remains valid if $\operatorname{char} k = p > 0$ and p doesn't divide $mn\prod_{1\le i\le l} m_i n_i$.

(2) Similarly, if p is a prime number and the affine curve is defined by $y^p = \prod_{1\le i\le l}(x-\lambda_i)^{m_i}$ such that the λ_i are distinct, $1 \le m_i < p$ and p doesn't divide $\sum_{1\le i\le l} m_i$, then Theorem 2 (and its proof for this case) remains valid no matter what $\operatorname{char} k$ may be. Note that the latter assumption can always be achieved. For, if we denote $\sum_{1\le i\le l} m_i$ by m and suppose that $m = pr$, we may assume that $\lambda_1 = 0$. Divide both sides of the equation by x^m. Consider the new variables $u = 1/x$, $v = y/x^r$.

(3) On the other hand, if we assume that k is a perfect field (such that (i) $p \nmid mn\prod_{1\le i\le l} m_i n_i$ if $\operatorname{char} k = p > 0$, or (ii) p is a prime number and the affine curve is defined by $y^p = \prod_{1\le i\le l}(x-\lambda_i)^{m_i}$ with ...) but not algebraically closed, then Theorem 2 is true because we can extend the constant field k to its algebraic closure at the beginning of the proof without affecting the genus by [Ch], p. 99.

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