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must be equivalent to one of a finite number of forms $r_i h_i$. Now let q be the least common denominator of the coefficients of $r_i h_i$. If we could have $f'(x_1, x_2, \ldots, x_{n-1}, 0)$ equivalent to $r_i h_i$ for an infinite number of possible values of α_2, with $0 \leq \alpha_2 < 1$, there would be two allowable values, say β and γ, with $0 < |\beta - \gamma| < \frac{1}{2q}$. Then considering $f'(0, 1, 0, \ldots, 0)$, we see that $f'(x_1, x_2, \ldots, x_{n-1}, 0)$ represents $k - \beta^2$ for $\alpha_2 = \beta$ and represents $k - \gamma^2$ for $\alpha_2 = \gamma$. However

$$\left|(k - \beta^2) - (k - \gamma^2)\right| = \left|\beta^2 - \gamma^2\right| < \frac{1}{2q}|\beta + \gamma| < \frac{1}{q}$$

contradicting the fact that distinct values of $r_i h_i$ are never closer than $\frac{1}{q}$. Similar considerations of $f'(0, 0, 1, \ldots, 0), \ldots, f'(0, 0, \ldots, 1, 0)$ show that there are only a finite number of allowable values of $\alpha_3, \ldots, \alpha_{n-1}$ for each $r_i h_i$.

Finally, to show that the number of allowable values of α_n in (8) is finite, consider the indefinite $(n-1)$-ary sections $f'(x_1, x_2, \ldots, x_{n-2}, 0, x_n)$, $f'(x_1, x_2, \ldots, x_{n-2}, x_n, x_n)$ and $f'(x_1, x_2, \ldots, x_{n-2}, 2x_n, x_n)$. At least one of these, called $\psi(x_1, x_2, \ldots, x_{n-2}, x_n)$ say, has a non-zero determinant (whose value depends only on k and the coefficients of g'). So, taking $\varepsilon_2 = \varepsilon\,|D(f)|^{1/n}\,|D(\psi)|^{-1/n-1} > 0$, ψ will represent a small positive value

$$v_2 < \varepsilon_2\,|D(\psi)|^{1/n-1} = \varepsilon\,|D(f)|^{1/n}$$

unless it is equivalent to a multiple of one of a finite number of forms. Since $N(\psi) = 1$, there will again only be one allowable multiple for each of the finite number of forms. As before, we can also see that for each of the finite number of possibilities for $\alpha_2, \ldots, \alpha_{n-1}, k, g'$ there will only be a finite number of allowable values of α_n.

It follows that the number of forms $f \in \mathcal{E}_n$ for which (6) fails (for a given $\varepsilon > 0$) is finite. So the theorem holds for n–ary forms.

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