Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 52 (2006)

Heft: 3-4: L'enseignement mathématique

Artikel: Hyperbolic polygons and simple closed geodesics

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Bibliographie

DOI: https://doi.org/10.5169/seals-2237

In [10], A. Yamada proved that $r \geq \ln \frac{2+\sqrt{7}}{\sqrt{3}}$ with equality occurring when S is the thrice punctured sphere (which proves that Yamada's lower bound is sharp). Interestingly, $\ln \frac{2+\sqrt{7}}{\sqrt{3}} = \operatorname{arcsinh}(\frac{2}{\sqrt{3}})$ and $\frac{1}{2}\ln 3 = \operatorname{arccosh}(\frac{2}{\sqrt{3}})$. The link is stronger than this apparent coincidence.

First of all, a thrice punctured sphere can be constructed by pasting two ideal hyperbolic triangles along all three edges. The three punctures are the points at infinity. The value $\frac{1}{2}\ln 3$ was also obtained using this triangle as the maximal radius of an inscribed disk. Theorem 5.6, seen in the light of Theorem 6.1, reads as follows:

THEOREM 6.2. *Let S be a hyperbolic Riemann surface of signature (g, n). There exists a point $x \in S$ such that $D(x, \frac{1}{2}\ln 3)$ is simply connected. The value $\frac{1}{2}\ln 3$ is sharp.* □

Notice that in contrast to Theorem 5.6, the value $\frac{1}{2}\ln 3$ is sharp for the set of surfaces with boundary but not for any individual surface. Surfaces with boundary play an important role in a variety of subjects, including the study of Klein surfaces (orientable or non-orientable hyperbolic surfaces). In other words, a Klein surface is either a hyperbolic Riemann surface, or the quotient of a closed hyperbolic Riemann surface by an orientation reversing involution (whose fixed point set is a set of disjoint simple closed geodesics). In terms of Klein surfaces, Theorem 5.6 implies the following corollary, where again the adjective "sharp" means sharp for the set of Klein surfaces.

COROLLARY 6.3. *Let S be a hyperbolic Klein surface. There exists a point $x \in S$ such that $D(x, \frac{1}{2}\ln 3)$ is simply connected. The value $\frac{1}{2}\ln 3$ is sharp.*

ACKNOWLEDGEMENTS. I thank Peter Buser for careful rereading and suggestions. I thank all the people I have bored with my ideas, and in particular Aline Aigon-Dupuy, who also helped with figures, and Gérard Maze.

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(Reçu le 14 juillet 2006)

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