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## 26

### ISOMORPHISM CONJECTURES FOR THE MAPPING CLASS GROUP

by James F. DAVIS

I enjoyed Guido's lecture in Münster where he showed that Teichmüller space is a classifying space for actions of the mapping class group whose isotropy groups are all finite. I will pose a question and then recall one I posed at that time.

Let  $\Sigma_g$  be a closed surface of genus  $g$ , let  $\Gamma_g$  be its mapping class group, and  $\tau_g \cong \mathbf{R}^{6g-6}$  its Teichmüller space.

QUESTION 26.1 (A special case of the Borel Conjecture). *Let  $\Gamma$  be a torsion free subgroup of the mapping class group, for example the Torelli group. Is the action of  $\Gamma$  on  $\tau_g$  topologically rigid? That is, is any proper homotopy equivalence of manifolds  $h: M \rightarrow \tau_g/\Gamma$ , which is a homeomorphism outside of a compact set, homotopic to a homeomorphism, via a homotopy which is constant outside of a compact set?*

QUESTION 26.2 (Isomorphism conjecture injectivity). *Is there a contractible compactification of Teichmüller space which is small at infinity and equivariant with respect to the action of the mapping class group?*

To say a compactification  $\bar{\tau}_g$  of  $\tau_g$  is *small at infinity* means that for every compact subset  $K$  of Teichmüller space, for every boundary point  $y \in \bar{\tau}_g - \tau_g$ , and for every neighborhood  $U$  of  $y$  in the compactification  $\bar{\tau}_g$ , there is a smaller neighborhood  $V$  so that for every  $\gamma \in \Gamma_g$ , if  $\gamma K \cap V$  is nonempty, then  $\gamma K \subset U$ .

DISCUSSION. A solution to Question 26.1 would likely involve carrying out the program of Farrell–Jones [1] in the mapping class group case. A positive solution to Question 26.2 would likely lead to a proof of the injectivity map

of the assembly map in  $K$ - and  $L$ -theory with respect to the family of finite subgroups (by modifying Rosenthal's thesis [3]), and thereby a new proof of the Novikov conjecture in this case. (It seems that the Novikov conjecture in the mapping class group case has been recently proved by Ursula Hamenstädt [2].)

## REFERENCES

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