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PROPER ACTIONS ON ACYCLIC SPACES

by Ian J. LEARY

Here are a few questions about proper cellular actions of discrete groups G on acyclic spaces. I have deliberately avoided the classifying space for proper G -actions, $\underline{E}G$, partly because some of the questions have already been answered for this space, and partly because I know that some other people will write in this volume about questions concerning $\underline{E}G$. I start with a version of the classic question that was posed by Ken S. Brown [1], p.226.

QUESTION 48.1. *If G is of finite virtual cohomological dimension, does G act properly on some acyclic space of dimension equal to $\text{vcd } G$?*

REMARK 48.2. If $\text{vcd } G$ is not equal to 2, then ‘acyclic’ in the above question can be replaced by ‘contractible’ without changing the question. The answer is ‘yes’ when $\text{vcd } G = 1$ by a theorem of Martin Dunwoody [2], and Quillen’s plus construction can be used to replace an acyclic space of dimension n by a contractible space of dimension equal to the maximum of n and 3.

Brita Nucinkis and I found examples to show that the dimension of the space $\underline{E}G$ can be strictly greater than $\text{vcd } G$ [5]. Some of the techniques that we used in [5], including Bredon cohomology, were learned from Guido Mislin.

Secondly, a rather vague question. It is well known that $\text{vcd } G$ is finite if and only if G is virtually torsion-free and G acts properly on some finite dimensional contractible space [1].

QUESTION 48.3. *Are there any results concerning group cohomology where virtual torsion-freeness plays a role? For example, are there any results about $H^*(G; \mathbf{Z}G)$ that hold for groups of finite vcd, but do not hold for all groups in Peter Kropholler's class $\mathfrak{H}_1\mathfrak{F}$?*

Peter Kropholler's class $\mathfrak{H}_1\mathfrak{F}$ consists of the groups that admit a proper action on some finite-dimensional contractible CW-complex. (See [3] for further details and for the definition of the larger class $\mathfrak{H}\mathfrak{F}$.)

Finally, a few questions concerning the connection between algebraic and topological finiteness conditions. See also [4], [5].

QUESTION 48.4. *If G is of type FP over a ring R , does G act cellularly cocompactly on some R -acyclic CW-complex X with stabilizers whose orders are units in R ?*

There is an algebraic version of this question too. Define a *projective permutation module* for the group algebra RG to be a direct sum of modules isomorphic to RG/H , where H ranges over the finite subgroups whose orders are units in R . Say that G is of type FPP over R if there is a finite resolution of R over RG by finitely generated projective permutation modules.

QUESTION 48.5. *If G is FP over R , is G necessarily of type FPP over R ?*

For $R = \mathbf{Z}$, this question is equivalent to the famous question of whether every group of type FP is FL.

QUESTION 48.6. *If G is FL over a prime field F , does G act freely cellularly cocompactly on some F -acyclic CW-complex?*

REMARK 48.7. There are groups that are FP but not FL over \mathbf{Q} , and are FL over \mathbf{C} [4].

Such a group cannot act freely cellularly cocompactly on any \mathbf{C} -acyclic CW-complex. It is because of these examples that the previous question is stated only for the fields \mathbf{Q} and \mathbf{F}_p .

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