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any correct expression will work. Now, using an argument of Humphries [9], for  $1 \leq i \leq g-2$  we can express  $T_{\delta_{i+2}}$  as a complicated product of elements in

$$\{T_{\alpha_i}, T_{\alpha_{i+1}}, T_{\alpha_{i+2}}, T_{\beta_i}, T_{\beta_{i+1}}, T_{\delta_i}, T_{\delta_{i+1}}\}^{\pm 1}.$$

This allows us eliminate  $T_{\delta_i}$  from  $S$  for  $i \geq 3$  by adding relations which do not involve both  $T_{\beta_1}^{\pm 1}$  and  $T_{\beta_{g-1}}^{\pm 1}$ . Our final relation is  $[h, T_{\delta_g}] = 1$ ; since this does not involve either  $T_{\beta_1}$  or  $T_{\beta_{g-1}}$ , we are done.  $\square$

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#### REFERENCES

- [1] BIRMAN, J. S. and B. WAJNRYB. Presentations of the mapping class group. Errata: “3-fold branched coverings and the mapping class group of a surface”. In: *Geometry and Topology (College Park, MD, 1983/84)*, 24–46. Lecture Notes in Mathematics 1167. Springer, Berlin, 1985. And by WAJNRYB: “A simple presentation of the mapping class group of an orientable surface”. *Israel J. Math.* 45 (1983), 157–174. Errata in: *Israel J. Math.* 88 (1994), 425–427.
- [2] BOWDITCH, B. H. and D. B. A. EPSTEIN. Natural triangulations associated to a surface. *Topology* 27 (1988), 91–117.
- [3] FARB, B. and N. V. IVANOV. The Torelli geometry and its applications: research announcement. *Math. Res. Lett.* 12 (2005), 293–301.
- [4] HARER, J. L. The virtual cohomological dimension of the mapping class group of an orientable surface. *Invent. Math.* 84 (1986), 157–176.
- [5] HARVEY, W. J. Geometric structure of surface mapping class groups. In: *Homological Group Theory (Proc. Sympos., Durham, 1977)*, 255–269. Cambridge Univ. Press, Cambridge, 1979.
- [6] HATCHER, A. On triangulations of surfaces. *Topology Appl.* 40 (1991), 189–194.
- [7] HATCHER, A. and W. THURSTON. A presentation for the mapping class group of a closed orientable surface. *Topology* 19 (1980), 221–237.
- [8] HATCHER, A. and K. VOGTMANN. Personal communication.
- [9] HUMPHRIES, S. P. Generators for the mapping class group. In: *Topology of Low-Dimensional Manifolds* (Proc. Second Sussex Conf., Chelwood Gate, 1977), 44–47. Lecture Notes in Mathematics 722. Springer, Berlin, 1979.

- [10] IVANOV, N. V. Complexes of curves and Teichmüller modular groups. *Uspekhi Mat. Nauk* 42 (1987), 49–91.
- [11] ——— Mapping class groups. In: *Handbook of Geometric Topology*, 523–633. North-Holland, Amsterdam, 2002.
- [12] JOHNSON, D. The structure of the Torelli group. I. A finite set of generators for  $\mathcal{I}$ . *Ann. of Math. (2)* 118 (1983), 423–442.
- [13] LICKORISH, W. B. R. A representation of orientable combinatorial 3-manifolds. *Ann. of Math. (2)* 76 (1962), 531–540.
- [14] MASUR, H. and S. SCHLEIMER. The pants complex has only one end. In: *Spaces of Kleinian Groups*, 209–218. Cambridge Univ. Press, Cambridge, 2006.
- [15] MCCARTHY, J. and W. VAUTAW. Automorphisms of Torelli groups. Preprint 2003.
- [16] MCCULLOUGH, D. and A. MILLER. The genus 2 Torelli group is not finitely generated. *Topology Appl.* 22 (1986), 43–49.
- [17] PENNER, R. C. The decorated Teichmüller space of punctured surfaces. *Comm. Math. Phys.* 113 (1987), 299–339.
- [18] PUTMAN, A. Cutting and pasting in the Torelli group. *Geom. Topol.* 11 (2007), 829–865.
- [19] SCHLEIMER, S. Notes on the complex of curves. Unpublished notes.
- [20] WAJNRYB, B. A simple presentation for the mapping class group of an orientable surface. *Israel J. Math.* 45 (1983), 157–174.
- [21] ——— An elementary approach to the mapping class group of a surface. *Geom. Topol.* 3 (1999), 405–466 (electronic).

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