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Autor: Baumslag, Gilbert
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GROUPS WITH THE SAME LOWER CENTRAL SEQUENCES

by Gilbert BAUMSLAG

Two groups G and H are said to have the same lower central sequences if

$$G/\gamma_n(G) \cong H/\gamma_n(H)$$

for every n, where $\gamma_n(G)$ denotes the n^{th} term of the lower central series of G.

Suppose that G and H are residually nilpotent, i.e., suppose that the intersection of their lower central series is the identity. The basic question then is how much do two residually nilpotent groups with the same lower central series have in common? So, for example:

- If G and H are both finitely generated and one is finitely presented, is the other also finitely presented?
- If G and H are both finitely generated and one has finitely generated H_2 with integral coefficients, does the other?
- If G is finitely generated and has the same lower central series as a free group, is $H_2(G, \mathbf{Z}) = 0$? Such a G is a so-called *parafree group* (see [1], [2]). This question has been tackled by many people and an incorrect proof has even been published. Bousfield and Kan [3] have proved that the pronilpotent completion of a residually nilpotent group G has the same lower central sequence as G. These completions turn up in homotopy theory, one of Guido's interests. However they do not, for the most part, reflect the properties of a given residually nilpotent group. It should be noted that the pronilpotent completion of a finitely generated, residually nilpotent group is finitely generated only if the group itself is nilpotent. In the case of a non–abelian, finitely generated free group, Bousfield and Kan have shown that the second homology group with integral coefficients of its pronilpotent completion has as many elements as the reals. So it is definitely not 0.

- If G and H are finitely generated nilpotent groups and have the same finite images, do they have the same homology?

The last of these questions is especially formulated for Guido who has been interested from time to time in the so-called genus of finitely generated nilpotent groups.

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Gilbert Baumslag

The CUNY Graduate Center
365 Fifth Avenue, Room 4208
New York, NY 10016-4309
USA
e-mail: gilbert.baumslag@gmail.com