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CS_n is the centralizer of $GL_k(\mathbf{C})$ on $W^{\otimes n}$, for $k \geq n$, where $W = \mathbf{C}^k$ is the representation of $GL_k(\mathbf{C})$ by matrix multiplication on column vectors. If we replace a simple tensor with the subset indexed by the binary string in its subscripts — for example $v_1 \otimes v_0 \otimes v_1 \otimes v_1 \otimes v_0 \Leftrightarrow 10110 \Leftrightarrow \{1, 3, 4\}$ — then the action on simple tensors is the same as the action of P_n on subsets in Section 2.

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REFERENCES

- [BQ] BENJAMIN, A.T. and J.J. QUINN. *Proofs that Really Count: the Art of Combinatorial Proof*. The Dolciani Mathematical Expositions 27. Mathematical Association of America, Washington DC, 2003.
- [CFS] CARTER, J.S., D. FLATH and M. SAITO. *The Classical and Quantum 6j-Symbols*. Mathematical Notes 43. Princeton University Press, Princeton, 1995.
- [CR] CURTIS, C.W. and I. REINER. *Representation Theory of Finite Groups and Associative Algebras*. Pure and Applied Mathematics 11. Wiley, New York, 1962.
- [GHJ] GOODMAN, F.M., P. DE LA HARPE and V.F.R. JONES. *Coxeter Graphs and Towers of Algebras*. Mathematical Sciences Research Institute Publications 14. Springer-Verlag, New York, 1989.
- [GL] GRAHAM, J.J. and G.I. LEHRER. Cellular algebras and diagram algebras in representation theory. In: *Representation Theory of Algebraic Groups and Quantum Groups*, 141–173. Advanced Studies in Pure Mathematics 40. Math. Soc. Japan, Tokyo, 2004.
- [Gr] GROOD, C. A Specht module analog for the rook monoid. *Electron. J. Combin.* 9 (2002), R2.
- [Ha] HALVERSON, T. Representations of the q -rook monoid. *J. Algebra* 273 (2004), 227–251.
- [HL] HALVERSON, T. and T. LEWANDOWSKI. RSK insertion for set partitions and diagram algebras. *Electron. J. Combin.* 11 (2004/06), R24.
- [HR] HALVERSON, T. and A. RAM. Partition algebras. *European J. Combin.* 26 (2005), 869–921.
- [Jo] JONES, V.F.R. Planar algebras, I. Preprint arXiv:math.QA/9909027 (1999).
- [Re] RENNER, L.E. *Linear Algebraic Monoids*. Encyclopaedia of Mathematical Sciences 134. Invariant Theory and Algebraic Transformation Groups, V. Springer-Verlag, Berlin, 2005.

- [Ri] RIORDAN, J. *Combinatorial Identities*. John Wiley & Sons, Inc., New York-London-Sydney, 1968.
- [OEIS] SLOANE, N. J. A. The On-Line Encyclopedia of Integer Sequences (2006), <http://www.research.att.com/~njas/sequences/A002720>.
- [So] SOLOMON, L. Representations of the rook monoid. *J. Algebra* 256 (2002), 309–342.

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