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**Autor:** Clark, Pete L.

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closure of  $S$  in the separable quadratic field extension  $k(E)/k(x)$ , it suffices to establish the following simple result.

LEMMA 17. *Let  $L/K$  be a finite Galois extension of fields, and  $S$  a Dedekind domain with fraction field  $L$ . Suppose that for all  $\sigma \in \text{Gal}(L/K)$ ,  $\sigma(S) = S$ . Then  $S$  is the integral closure of  $R := S \cap K$  in  $L$ .*

*Proof.* Since  $S$  is integrally closed, it certainly contains the integral closure of  $R$  in  $L$ . Conversely, for any  $x \in S$ ,  $P(t) = \prod_{\sigma \in \text{Gal}(L/K)} (t - \sigma(x))$  is a monic polynomial with coefficients in  $(S \cap K)[t]$  satisfied by  $x$ .

REMARK. It is possible to avoid the use of an elliptic curve with trivial Mordell-Weil group: since we are, in general, passing to a quotient anyway, we can just mod out by  $E(k)$ . In fact, at the expense of introducing minor complications, one can make the argument go through starting with any elliptic curve  $E$  over any field  $k$  whatsoever.

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Pete L. Clark

Department of Mathematics  
Boyd Graduate Studies Research Center  
University of Georgia  
Athens, GA 30602-7403  
U. S. A.  
*e-mail* : pete@math.uga.edu