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closure of S in the separable quadratic field extension $k(E)/k(x)$, it suffices to establish the following simple result.

LEMMA 17. *Let L/K be a finite Galois extension of fields, and S a Dedekind domain with fraction field L. Suppose that for all $\sigma \in \mathrm{Gal}(L/K)$, $\sigma(S) = S$. Then S is the integral closure of $R := S \cap K$ in L.*

Proof. Since S is integrally closed, it certainly contains the integral closure of R in L. Conversely, for any $x \in S$, $P(t) = \prod_{\sigma \in \mathrm{Gal}(L/K)} \big(t - \sigma(x)\big)$ is a monic polynomial with coefficients in $(S \cap K)[t]$ satisfied by x.

REMARK. It is possible to avoid the use of an elliptic curve with trivial Mordell-Weil group: since we are, in general, passing to a quotient anyway, we can just mod out by $E(k)$. In fact, at the expense of introducing minor complications, one can make the argument go through starting with any elliptic curve E over any field k whatsoever.

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