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Remarks Concerning a Paper by Wilker[1])

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WILKER states that, $W(1)^2$,

$$\frac{dr}{dt} = \frac{\partial r}{\partial t} + [r, H] \tag{1}$$

is invariant with respect to a homogeneous canonical transformation only for the trivial case of a free particle.

WILKER deduces this result by using the homogeneous canonical formalism, $W(2)$, $W(3)$,

$$q_i' = \frac{\partial \mathfrak{H}}{\partial p_i}, \quad P_i' = -\frac{\partial \mathfrak{H}}{\partial q_i}; \quad \mathfrak{H}(q_i, p_i) = 0 \tag{2}$$

$$[i = 1, \cdots f+1]$$

$$r' = \{r, \mathfrak{H}\} \tag{3}$$

$$\mathfrak{H} \equiv t'(\tau)\,(H + p_t) \tag{4}$$

WILKER shows, $W(4)$, that, if $r = r\,(q_i, t, p_i, p_t)$

$$\frac{dr}{dt} = \frac{\partial r}{\partial t} + [r, H] - \frac{\partial H}{\partial t}\,\frac{\partial r}{\partial p_t} \tag{5}$$

WILKER now concludes that (1) follows from (5) only if $\partial H/\partial t = 0$.

Now (1) is usually understood to apply in the formalism that can be LORENTZ invariant, but that is not LORENTZ invariant in appearance. In this formalism r is expressed as a function of q_i, p_i, t. If r is a function of H as well (or any other quantity, e.g. angular momentum),

$$\frac{dr}{dt} = \frac{\partial r}{\partial t} + [r, H] + \frac{\partial r}{\partial H}\,\frac{\partial H}{\partial t} \tag{6}$$

where $dH/dt = \partial H/\partial t$.

(6) is equivalent to WILKER's $W(4)$, (5) above, because $\mathfrak{H} = 0$, and $p_t = -H$. However, the third term on the right side of (6) will

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[1]) P. WILKER, Helv. Phys. Acta, 319, **24**, 1951.

[2]) $W(1)$ etc. refers to WILKER's paper.

not appear if H is expressed as a function of q_i, p_i, t. (6) therefore reduces to (1) by arranging that $\partial r/\partial H = 0$, and not by using WILKER's stronger condition $\partial H/\partial t = 0$.

If $r = r(q_i, p_i, t)$ in one inertial system, and we carry out a LORENTZ transformation, the Hamiltonian will appear in the function. For a scalar r

$$r(q_i, p_i, t) = \overline{r}(\overline{q}_i, \overline{p}_i, \overline{H}, \overline{t}) \tag{7}$$

where the bar quantities refer to the new inertial system. $\overline{H}$, however, can be expressed as a function of $\overline{q}_i$, $\overline{p}_i$, $\overline{t}$, and, if we substitute $\overline{H}$ expressed as a function of $\overline{q}_i$, $\overline{p}_i$, $\overline{t}$ in $\overline{r}$, the expression (1) will also hold in the new inertial system, because $\partial \overline{r}/\partial \overline{H} = 0$.

It may also be remarked here that the homogeneous canonical formalism can be developed very simply by relating the formulation that is non-covariant in appearance

$$\delta \int L\left(x_i, \frac{dx_i}{dt}, t\right) dt = 0$$

with the covariant formulation

$$\delta \int \mathfrak{L}\left(x_\mu, \frac{dx_\mu}{d\tau}\right) d\tau = 0$$

in the following way:

$$L\left(x_i, \frac{dx_i}{dt}, t\right) dt = \mathfrak{L}\left(x_\mu, \frac{dx_\mu}{d\tau}\right) d\tau. \tag{8}$$