

**Zeitschrift:** Helvetica Physica Acta  
**Band:** 25 (1952)  
**Heft:** III

**Artikel:** Remarks Concerning a Paper by Wilker  
**Autor:** Iskraut, Richard W.  
**DOI:** <https://doi.org/10.5169/seals-112311>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

**Download PDF:** 01.04.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

## Remarks Concerning a Paper by Wilker<sup>1)</sup>

by Richard W. Iskraut

University of Maryland, College Park, Maryland, U.S.A.

(20. XII. 1951.)

WILKER states that,  $W(1)$ <sup>2)</sup>,

$$\frac{dr}{dt} = \frac{\partial r}{\partial t} + [r, H] \quad (1)$$

is invariant with respect to a homogeneous canonical transformation only for the trivial case of a free particle.

WILKER deduces this result by using the homogeneous canonical formalism,  $W(2)$ ,  $W(3)$ ,

$$q'_i = \frac{\partial \mathfrak{S}}{\partial p_i}, \quad P'_i = -\frac{\partial \mathfrak{S}}{\partial q_i}; \quad \mathfrak{S}(q_i, p_i) = 0 \quad (2)$$

$$[i = 1, \dots, f+1]$$

$$r' = \{r, \mathfrak{S}\} \quad (3)$$

$$\mathfrak{S} \equiv t'(\tau) (H + p_t) \quad (4)$$

WILKER shows,  $W(4)$ , that, if  $r = r(q_i, t, p_i, p_t)$

$$\frac{dr}{dt} = \frac{\partial r}{\partial t} + [r, H] - \frac{\partial H}{\partial t} \frac{\partial r}{\partial p_t} \quad (5)$$

WILKER now concludes that (1) follows from (5) only if  $\partial H/\partial t = 0$ .

Now (1) is usually understood to apply in the formalism that can be LORENTZ invariant, but that is not LORENTZ invariant in appearance. In this formalism  $r$  is expressed as a function of  $q_i$ ,  $p_i$ ,  $t$ . If  $r$  is a function of  $H$  as well (or any other quantity, e.g. angular momentum),

$$\frac{dr}{dt} = \frac{\partial r}{\partial t} + [r, H] + \frac{\partial r}{\partial H} \frac{\partial H}{\partial t} \quad (6)$$

where  $dH/dt = \partial H/\partial t$ .

(6) is equivalent to WILKER's  $W(4)$ , (5) above, because  $\mathfrak{S} = 0$ , and  $p_t = -H$ . However, the third term on the right side of (6) will

<sup>1)</sup> P. WILKER, *Helv. Phys. Acta*, 319, **24**, 1951.

<sup>2)</sup>  $W(1)$  etc. refers to WILKER's paper.

not appear if  $H$  is expressed as a function of  $q_i, p_i, t$ . (6) therefore reduces to (1) by arranging that  $\partial r/\partial H = 0$ , and not by using WILKER's stronger condition  $\partial H/\partial t = 0$ .

If  $r = r(q_i, p_i, t)$  in one inertial system, and we carry out a LORENTZ transformation, the Hamiltonian will appear in the function. For a scalar  $r$

$$r(q_i, p_i, t) = \bar{r}(\bar{q}_i, \bar{p}_i, \bar{H}, \bar{t}) \quad (7)$$

where the bar quantities refer to the new inertial system.  $\bar{H}$ , however, can be expressed as a function of  $\bar{q}_i, \bar{p}_i, \bar{t}$ , and, if we substitute  $\bar{H}$  expressed as a function of  $\bar{q}_i, \bar{p}_i, \bar{t}$  in  $\bar{r}$ , the expression (1) will also hold in the new inertial system, because  $\partial \bar{r}/\partial \bar{H} = 0$ .

It may also be remarked here that the homogeneous canonical formalism can be developed very simply by relating the formulation that is non-covariant in appearance

$$\delta \int L\left(x_i, \frac{dx_i}{dt}, t\right) dt = 0$$

with the covariant formulation

$$\delta \int \mathfrak{L}\left(x_\mu, \frac{dx_\mu}{d\tau}\right) d\tau = 0$$

in the following way:

$$L\left(x_i, \frac{dx_i}{dt}, t\right) dt = \mathfrak{L}\left(x_\mu, \frac{dx_\mu}{d\tau}\right) d\tau. \quad (8)$$