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Nuclear Shell Model and Pseudoscalar Potential*)

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Bleuler and Terreaux[1]) suggested in a recent article that it might be profitable to examine the properties of a particle (nucleon) obeying the Dirac equation in the field on an external ("classical") pseudoscalar potential. Their equation (9) is:

$$c\left(\frac{\hbar}{i}\,\frac{\partial}{\partial x_\mu}\,\gamma_\mu\right)\psi + mc^2\psi + V'\gamma_5\,\psi = 0\,. \qquad (1)$$

This, as is well known, is one of the special forms which the generalized Dirac equation with the famous five interactions can assume. It has been repeatedly proposed[2]) that one might substitute a pseudoscalar potential for either a scalar or a four-vector potential as a possible source of an effective spin-orbit interaction which one hoped would be large enough (and of the correct sign) for the requirements of nuclear levels in the single particle shell model.

Such a proposal faces the handicap that the Dirac equation with a *pure pseudoscalar potential admits no bound states*. The formal proof of this assertion was given by Pekar[3]), and it was found independently by the author. The proof is very simple and goes as follows:

V' must be imaginary if the Hamiltonian is to be Hermitian. Hence $V'^* = -V'$. For a stationary state the Dirac equation can be written as

$$(E\gamma_4 - mc^2)\psi = (i\vec{\gamma}\cdot\vec{p} + \gamma_5 V')\psi\,. \qquad (2)$$

The operator which acts on ψ on the left hand side of this equation is Hermitian, while the one on the right hand side is antihermitian. Hence the expectation value of either operator in the stationary state must vanish.

We conclude:

$$E(\psi, \gamma_4\psi) = mc^2(\psi, \psi)\,. \qquad (3)$$

γ_4 is unitary. Hence the Schwarz inequality gives

$$(\psi, \gamma_4\psi) \leqslant (\psi, \psi)$$

and consequently

$$E \geqslant mc^2.$$

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Thus there can be no bound states in the usual sense. For the special case of a $1/r$ potential PÉTIAU[2]) had already noted this fact.

One can see clearly what obstructs the formation of a bound state here, if the iterated second-order equation for ψ is constructed. It turns out to be:

$$[E^2 - (mc^2)^2]\,\psi = [-c^2\hbar^2\nabla^2 - V'^2 + i\hbar c\beta\vec{\sigma}\cdot\nabla V']\,\psi. \quad (4)$$

The last term on the right hand side is in the nature of a spin-orbit interaction, and its presence is usually considered as promising to provide the splitting of energy levels needed for the j — j coupling shell model. However, there is inevitably a repulsive $-V'^2$ term which prevents binding. These features appear also in the non-relativistic approximation[4]).

The inability of the pseudoscalar potential to bind does, of course, not preclude its possible usefulness as an approximation in scattering problems[5]).

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