Zeitschrift: Helvetica Physica Acta
Band: 30 (1957)
Heft: I

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Autor: Holdaway, H.W.
DOI: https://doi.org/10.5169/seals-112805

A Note on Electromagnetic Flowmeters of Rectangular Cross-Section

by **H. W. Holdaway**
(N. S. W. University of Technology, Sydney, Australia.)

(13. IX. 1956.)

Summary. This paper is intended to supplement that of THÜRLEMANN [H.P.A. **28**, 483 (1955)]. It is demonstrated that, for two-dimensional flow distributions in a conduit subject to a uniform transverse magnetic field, the e. m. f. generated between two opposite conducting faces of the rectangular cross-section is proportional to the true average velocity of flow, *irrespective of the nature of the two dimensional distribution.* The form of analysis employed permits an examination also of the effect of internal resistance of the unit when connected to an external measuring circuit.

The most obvious advantage of a rectangular cross-section for an electromagnetic flowmeter is that the geometry is well adapted to use with practical magnet structures, and that within certain limits it permits a reduction in the spacing of pole faces and an increase in electrode spacing, resulting in somewhat higher induced e.m.f.'s for a given cross-sectional area and mean velocity of flow.

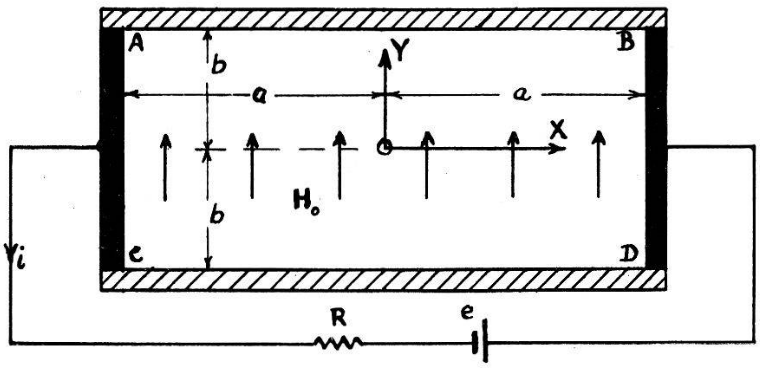

Fig. 1.
Flowmeter Cross-Section (Schematic).

However, as the following analysis shows, the rectangular cross-section leads to an induced e.m.f. which is proportional to the true mean velocity of flow (or total discharge) in the case of an established flow regime in a uniform magnetic field.

It is assumed that the magnetic field $\bar{H}_0$ is uniform and in vector notation is equal to $H_0\,\hat{j}$, where $\hat{j}$ is the unit vector parallel to the Y axis. The fluid velocity $\bar{v}$ may be distributed in any arbitrary manner over the cross-section, but is assumed to be parallel to the Z axis and can be written $\bar{v} = v\,\hat{k}$, where $\hat{k}$ is a unit vector upwards in Fig. 1 and V is independent of Z. Two opposite sides of the cross-section are conducting and two insulating, as shown in Fig. 1.

The equations applicable to steady state conditions are (in electromagnetic units)

$$\bar{\nabla}^2 U = \bar{H}_0 \cdot \operatorname{curl} \bar{v} = -H_0 \frac{\partial v}{\partial x} \tag{1}$$

whilst at the boundries of the cross-section the current density is

$$\bar{J} = \sigma \bar{E} = -\sigma \bar{\nabla} U. \tag{2}$$

(Here $\bar{E}$ is electric intensity, U the electric potential, σ the specific conductivity of the fluid, all in electromagnetic units.)

Therefore the current leaving the fluid at a conducting face is, per unit length of flowmeter,

$$\int \bar{J} \cdot \bar{n}\, ds = -\sigma \int \frac{\partial U}{\partial n}\, ds \tag{3}$$

where ds is an element of the perimeter of the cross-section. Thus if l is the length of the measuring section of the flowmeter and i the external current,

$$\int \frac{\partial U}{\partial n}\, ds = \pm \frac{i}{l\sigma} \tag{4}$$

at a conducting face.

We now apply the 2-dimensional Green's Theorem

$$\int V \frac{\partial U}{\partial n\, ds} - \int U \frac{\partial V}{\partial n\, ds} = \int\!\!\int (V \bar{\nabla}^2 U - U \bar{\nabla}^2 V)\, dx\, dy \tag{5}$$

and chose $V = x$, so that $\bar{\nabla}^2 V$ vanishes identically. On the surface AC of Fig. 1.

$$\frac{\partial V}{\partial n} = -\frac{\partial V}{\partial x} = -1, \quad V = -a, \quad U = U_A. \tag{6}$$

On the surface BD

$$\frac{\partial V}{\partial n} = \frac{\partial V}{\partial x} = 1, \quad V = a, \quad U = U_B. \tag{7}$$

On surfaces AB and CD

$$\frac{\partial V}{\partial n} = 0, \quad \frac{\partial U}{\partial n} = 0. \tag{8}$$

Substituting from equations (1), (6), (7) and (8) into equation (5) we obtain

$$-\int_{AB} a \frac{\partial U}{\partial n\, ds} + \int_{BD} a \frac{\partial U}{\partial n\, ds} + 2\,b(U_A - U_B) = -H_0 \int\int x \frac{\partial v}{\partial x}\, dx\, dy$$
$$= + H_0 \int\int v\, dx\, dy\,. \qquad (9)$$

The integral on the right hand side of equation (9) takes the form shown since $v = 0$ at all boundary surfaces. It can be expressed as

$$H_0 \int\int v\, dx\, dy = H_0\, v_m\, A = H_0\, Q \qquad (10)$$

where $A = 4\,ab$ is the cross-sectional area of the flowmeter, v_m is the true mean velocity and $Q = v_m A$ is the total flow or discharge.

For zero external current, equations (9) and (10) lead to the result

$$\boxed{U_A - U_B = \frac{H_0 Q}{2\,b} = 2\,a\,H_0\,v_m} \qquad (11)$$

and thus the potential difference is equal to that induced in a conductor of length equal to the electrode spacing, travelling across the field at the true mean velocity of the fluid.

If there is an external current i (we temporarily generalise by including an external battery of e. m. f. „e“) and the series resistance is R, then

$$i = \frac{U_A - U_B + e}{R}\,. \qquad (12)$$

On substituting from equation (4) into equation (9), the latter now reduces to

$$\boxed{\left(\frac{a\,i}{l\sigma} + \frac{a\,i}{l\sigma}\right) + 2\,b\,(U_A - U_B) = H_0\,Q} \qquad (13)$$

Combining equations (12) and (13) there results

$$(U_A - U_B)\left(1 + \frac{a}{Rb\,l\sigma}\right) + \frac{e\,a}{Rb\,l\sigma} = \frac{H_0\,Q}{2\,b}\,. \qquad (14)$$

Putting either $Q = 0$ or $H_0 = 0$ it is easily shown that the effective internal resistance of the flowmeter is

$$R_e = \frac{a}{b\,l\sigma}\,. \qquad (15)$$

Thus finally after putting $e = 0$

$$\boxed{U_A - U_B = \frac{H_0 Q}{2\,b}\left(\frac{R}{R + R_e}\right) = 2\,a\,H_0\,v_m\left(\frac{R}{R + R_e}\right)\,.} \qquad (16)$$

Equation (16) reduces to equation (11) when $R \gg R_e$. Equations (15) and (16) indicate the desirable magnitude of external resistance, or equation (13) the limiting magnitude of current for any desired degree of approximation to the ideal equation (11). In practice the maximum current may be restricted by other factors not considered here, such as current density limits needed to minimise polarisation effects or to ensure a close approach to ideal behaviour with reversible electrode systems.