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# Fourth order magnetic moment of the electron

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In connection with the upper and lower bounds analysis done by the author[1]), which indicated a clear discrepancy with the Karplus and Kroll's result for the 4th order magnetic moment[2]), we have performed an analytic evaluation of the five independent diagrams contributing to this moment in fourth order*). The results are the following:

$$\mu_{\mathrm{I}} = \frac{\alpha^2}{\pi^2}\left(\frac{1}{6} + \frac{13}{36}\pi^2 + \frac{5}{4}\zeta(3) - \frac{5}{6}\pi^2 \operatorname{Log} 2\right) = -0.467\frac{\alpha^2}{\pi^2}. \quad (1)$$

$$\mu_{\mathrm{II_a}} = \frac{\alpha^2}{\pi^2}\left(\frac{11}{48} + \frac{\pi^2}{18}\right) = 0.778\frac{\alpha^2}{\pi^2}. \quad (2)$$

$$\mu_{\mathrm{II_c}} = \frac{\alpha^2}{\pi^2}\left(-\frac{67}{24} + \frac{\pi^2}{18} - \frac{1}{2}\zeta(3) + \frac{1}{3}\pi^2 \operatorname{Log} 2 - \frac{1}{2}\operatorname{Log}\frac{\lambda^2}{m^2}\right) = -0.564\frac{\alpha^2}{\pi^2} - \frac{1}{2}\frac{\alpha^2}{\pi^2}\operatorname{Log}\frac{\lambda^2}{m^2}. \quad (3)$$

$$\mu_{\mathrm{II_d}} = \frac{\alpha^2}{\pi^2}\left(\frac{11}{24} - \frac{\pi^2}{18} + \frac{1}{2}\operatorname{Log}\frac{\lambda^2}{m^2}\right) = -0.090\frac{\alpha^2}{\pi^2} + \frac{1}{2}\frac{\alpha^2}{\pi^2}\operatorname{Log}\frac{\lambda^2}{m^2}. \quad (4)$$

$$\mu_{\mathrm{II_e}} = \frac{\alpha^2}{\pi^2}\left(\frac{119}{36} - \frac{\pi^2}{3}\right) = 0.016\frac{\alpha^2}{\pi^2}. \quad (5)$$

$$\mu^{(4)}_{\mathrm{total}} = \frac{\alpha^2}{\pi^2}\left(\frac{197}{144} + \frac{\pi^2}{12} + \frac{3}{4}\zeta(3) - \frac{1}{2}\pi^2 \operatorname{Log} 2\right) = -0.328\frac{\alpha^2}{\pi^2}. \quad (6)$$

Compared with the values given in their original paper by Karplus and Kroll, one can see that two terms were in error: $\mu_{\mathrm{I}}$ differs by

$$\frac{\alpha^2}{\pi^2}\frac{1}{32} = 0.031\frac{\alpha^2}{\pi^2};$$

$$\mu_{\mathrm{II_c}} \text{ by } \frac{\alpha^2}{\pi^2}\left(\frac{32}{3} - \frac{61}{8}\pi^2 + \frac{17}{2}\pi^2 \operatorname{Log} 2 - \frac{109}{4}\zeta(3)\right) = 2.614\frac{\alpha^2}{\pi^2}.$$

The three other terms check. The error in $\mu_{\mathrm{I}}$ remained of course undetected in the upper and lower bound analysis owing to its small-

*) The terminology of ref. 2 is used throughout this paper.

ness. But the large discrepancy in $\mu_{\mathrm{II}_c}$ was that pin-pointed out in the previous paper.

A summary of the most important electromagnetic observables, the theoretical values of which are modified by the new value of the magnetic moment, is now given:

Moment of the electron: $\frac{\mu_e}{\mu_0} = 1.0011596 = 1 + \frac{\alpha}{2\pi} - 0.328\,\frac{\alpha^2}{\pi^2}$.

Franken and Liebes' value for it: $\mu_e/\mu_0 = 1.001167 \pm 0.000005$*).

$g$-factor of the $\mu$-meson (electromagnetic):

$$2\,(1.0011654) = 2\left(1 + \frac{\alpha}{2\pi} + 0.75\,\frac{\alpha^2}{\pi^2}\right).$$

Last Lederman's value: 2 (1.0021 $\pm$ 0.0008) *).

$2^2\,S_{1/2} - 2^2\,P_{1/2}$ (Hydrogen): (1057.94 $\pm$ 0.15) Mc/s; observed: (1057·77 $\pm$ 0.10) Mc/s.

$2^2\,S_{1/2} - 2^2\,P_{1/2}$ (Deuterium): (1059.22 $\pm$ 0.15) Mc/s; observed: (1059.00 $\pm$ 0.10) Mc/s.

Fine structure constant: $1/\alpha = 137.0384$; (previously: 137.0365).

The new fourth order correction given here is in agreement with:

a) The upper and lower bounds given by the author[1]).

b) A calculation using a different method, performed by C. Sommerfield[3]).

c) A recalculation done by N. M. Kroll and collaborators*).

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## References.

1) A. Petermann. Nuclear Physics **3**, 689 (1957) and Nuclear Physics in the press.
2) R. Karplus and N. M. Kroll, Phys. Rev. **77**, 536 (1950).
3) C. Sommerfield, Phys. Rev. In the press.

*) Private Communication.