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## Fourth order magnetic moment of the electron

by **A. Petermann.**

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In connection with the upper and lower bounds analysis done by the author<sup>1)</sup>, which indicated a clear discrepancy with the Karplus and Kroll's result for the 4th order magnetic moment<sup>2)</sup>, we have performed an analytic evaluation of the five independent diagrams contributing to this moment in fourth order\*). The results are the following:

$$\mu_{\text{I}} = \frac{\alpha^2}{\pi^2} \left( \frac{1}{6} + \frac{13}{36} \pi^2 + \frac{5}{4} \zeta(3) - \frac{5}{6} \pi^2 \text{Log } 2 \right) = -0.467 \frac{\alpha^2}{\pi^2}. \quad (1)$$

$$\mu_{\text{II}_a} = \frac{\alpha^2}{\pi^2} \left( \frac{11}{48} + \frac{\pi^2}{18} \right) = 0.778 \frac{\alpha^2}{\pi^2}. \quad (2)$$

$$\begin{aligned} \mu_{\text{II}_c} = \frac{\alpha^2}{\pi^2} \left( -\frac{67}{24} + \frac{\pi^2}{18} - \frac{1}{2} \zeta(3) + \frac{1}{3} \pi^2 \text{Log } 2 - \frac{1}{2} \text{Log } \frac{\lambda^2}{m^2} \right) = \\ -0.564 \frac{\alpha^2}{\pi^2} - \frac{1}{2} \frac{\alpha^2}{\pi^2} \text{Log } \frac{\lambda^2}{m^2}. \end{aligned} \quad (3)$$

$$\mu_{\text{II}_d} = \frac{\alpha^2}{\pi^2} \left( \frac{11}{24} - \frac{\pi^2}{18} + \frac{1}{2} \text{Log } \frac{\lambda^2}{m^2} \right) = -0.090 \frac{\alpha^2}{\pi^2} + \frac{1}{2} \frac{\alpha^2}{\pi^2} \text{Log } \frac{\lambda^2}{m^2}. \quad (4)$$

$$\mu_{\text{II}_e} = \frac{\alpha^2}{\pi^2} \left( \frac{119}{36} - \frac{\pi^2}{3} \right) = 0.016 \frac{\alpha^2}{\pi^2}. \quad (5)$$

$$\mu_{\text{total}}^{(4)} = \frac{\alpha^2}{\pi^2} \left( \frac{197}{144} + \frac{\pi^2}{12} + \frac{3}{4} \zeta(3) - \frac{1}{2} \pi^2 \text{Log } 2 \right) = -0.328 \frac{\alpha^2}{\pi^2}. \quad (6)$$

Compared with the values given in their original paper by KARPLUS and KROLL, one can see that two terms were in error:  $\mu_{\text{I}}$  differs by

$$\frac{\alpha^2}{\pi^2} \frac{1}{32} = 0.031 \frac{\alpha^2}{\pi^2};$$

$$\mu_{\text{II}_c} \text{ by } \frac{\alpha^2}{\pi^2} \left( \frac{32}{3} - \frac{61}{8} \pi^2 + \frac{17}{2} \pi^2 \text{Log } 2 - \frac{109}{4} \zeta(3) \right) = 2.614 \frac{\alpha^2}{\pi^2}.$$

The three other terms check. The error in  $\mu_{\text{I}}$  remained of course undetected in the upper and lower bound analysis owing to its small-

\*) The terminology of ref. 2 is used throughout this paper.

ness. But the large discrepancy in  $\mu_{\text{IIc}}$  was that pin-pointed out in the previous paper.

A summary of the most important electromagnetic observables, the theoretical values of which are modified by the new value of the magnetic moment, is now given:

$$\text{Moment of the electron: } \frac{\mu_e}{\mu_0} = 1.0011596 = 1 + \frac{\alpha}{2\pi} - 0.328 \frac{\alpha^2}{\pi^2}.$$

FRANKEN and LIEBES' value for it:  $\mu_e/\mu_0 = 1.001167 \pm 0.000005^*$ .  
 $g$ -factor of the  $\mu$ -meson (electromagnetic):

$$2 (1.0011654) = 2 \left( 1 + \frac{\alpha}{2\pi} + 0.75 \frac{\alpha^2}{\pi^2} \right).$$

Last Lederman's value:  $2 (1.0021 \pm 0.0008)^*$ .

$$2^2 S_{1/2} - 2^2 P_{1/2} \text{ (Hydrogen): } (1057.94 \pm 0.15) \text{ Mc/s; observed: } (1057.77 \pm 0.10) \text{ Mc/s.}$$

$$2^2 S_{1/2} - 2^2 P_{1/2} \text{ (Deuterium): } (1059.22 \pm 0.15) \text{ Mc/s; observed: } (1059.00 \pm 0.10) \text{ Mc/s.}$$

Fine structure constant:  $1/\alpha = 137.0384$ ; (previously:  $137.0365$ ).

The new fourth order correction given here is in agreement with:

- a) The upper and lower bounds given by the author<sup>1</sup>).
- b) A calculation using a different method, performed by C. SOMMERFIELD<sup>3</sup>).
- c) A recalculation done by N. M. KROLL and collaborators\*).

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#### References.

- 1) A. PETERMANN. Nuclear Physics **3**, 689 (1957) and Nuclear Physics in the press.
- 2) R. KARPLUS and N. M. KROLL, Phys. Rev. **77**, 536 (1950).
- 3) C. SOMMERFIELD, Phys. Rev. In the press.

\*) Private Communication.