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Necessary Restriction on Wightman Functions*)

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Wightman has translated his axioms of field theory into properties of the vacuum expectation values of field operators (Wightman functions)¹⁾.

We will, as usual, illustrate the following remarks by the theory of a real scalar field $A(x)$; then these functions take the form

$$\mathfrak{W}_n(x_1, x_2, \dots, x_n) = (\Omega, A(x_1) A(x_2) \dots A(x_n) \Omega).$$

The necessary conditions for a sequence of such functions to correspond to the theory of a scalar field naturally decompose into linear and non-linear restrictions. It seems that one essential non-linear restriction has been overlooked till now. It is the purpose of this letter to emphasize this additional condition. It has to do with the uniqueness of the vacuum state Ω .

We need the following axioms explicitly:

1. The translational invariance and the existence of a unitary translation operator $T(a)$ defined by $T(a) \Omega = \Omega$, $T(a) A(x) T^*(a) = A(x + a)$.
2. The spectrum conditions for the energy momentum vector P_μ defined by $T(a) = \exp i(P_\mu a^\mu)$: The spectrum of P has an isolated simple eigenvalue at $p_\mu = 0$, corresponding to the eigenvector Ω . All other points of the spectrum are in the forward cone.

The second axiom has the following consequence: let $\chi(a) = (2\pi)^{-4} \int e^{i(p, a)} \tilde{\chi}(p) d^4p$ be a testing function and let the support of $\tilde{\chi}(p)$ contain only the point $p = 0$ of the energy momentum spectrum. Then $\int \chi(a) T(a) d^4a = \tilde{\chi}(0) E_\Omega$ where E_Ω is the projection on the vacuum state Ω .

*) Die vorliegende kurze Note war als Mitteilung in Phys. Rev. Letters gedacht. Sie teilte aber das Schicksal der grundlegenden Arbeit von HALL und WIGHTMAN beim Phys. Rev. und gelangt daher in ihrer ursprünglichen Form hier zum Abdruck. R. J.

This has as consequence for the Wightman functions

$$\begin{aligned} \int \chi(a) \mathfrak{W}_n(x_1, \dots, x_{k-1}, x_k + a, x_{k+1} + a, \dots, x_n + a) d^4a = \\ = \tilde{\chi}(0) \mathfrak{W}_{k-1}(x_1, \dots, x_{k-1}) \mathfrak{W}_{n-k+1}(x_k, \dots, x_n). \end{aligned} \tag{1}$$

Proof

$$\begin{aligned} \int \chi(a) \mathfrak{W}(x_1, \dots, x_{k-1}, x_k + a, \dots, x_n + a) d^4a = \\ = (\Omega, A(x_1) \dots A(x_{k-1}) \int \chi(a) T(a) d^4a A(x_k) \dots A(x_n) \Omega) = \\ = (\Omega, A(x_1) \dots A(x_{k-1}) \Omega) (\Omega, A(x_k) \dots A(x_n) \Omega). \end{aligned}$$

Equation (1) is essentially the additional condition mentioned above.

In a more technical way the following can be easily shown: If a sequence of (tempered) distributions $\mathfrak{W}_n(x_1, \dots, x_n)$ satisfies all the Wightman properties and in addition the equations

$$\begin{aligned} \int \varphi^*(x_1, \dots, x_n) \mathfrak{W}_{2n}(x_n, \dots, x_1, y_1, \dots, y_n) \varphi(y_1, \dots, y_n) d^{4n}x d^{4n}y = \\ = \left| \int \mathfrak{W}_n(x_1, \dots, x_n) \varphi(x_1, \dots, x_n) d^{4n}x \right|^2 \end{aligned} \tag{2}$$

for all testing functions $\varphi(x_1, \dots, x_n)$ of which the Fouriertransform $\tilde{\varphi}(p_1, \dots, p_n)$ vanishes if $p_1 + p_2 + \dots + p_n \neq 0$ is in the spectrum of P , then these \mathfrak{W}_n are Wightman functions to a field $A(x)$, which satisfies all the Wightman axioms. Our condition (2) insures the uniqueness of the vacuum.

Remarks

1. The condition (2) is seemingly weaker than (1). It however insures (together with the Wightman-conditions) the validity of (1).
2. The condition (2) is independent of the Wightman conditions. In order to see this we remark that from two different sequences $\mathfrak{W}_n^{(1)}$ and $\mathfrak{W}_n^{(2)}$ satisfying the Wightman conditions we can construct a third sequence $\mathfrak{W}_n = \alpha \mathfrak{W}_n^{(1)} + (1 - \alpha) \mathfrak{W}_n^{(2)}$, also satisfying the same conditions, provided $0 \leq \alpha \leq 1$. If we assume now that (2) is also satisfied, then (1) can be used for $\mathfrak{W}_n, \mathfrak{W}_n^{(1)}$ and $\mathfrak{W}_n^{(2)}$ and leads (provided that $\tilde{\chi}(0) \neq 0$) to

$$\begin{aligned} (\alpha \mathfrak{W}_k^{(1)} + (1 - \alpha) \mathfrak{W}_k^{(2)}) (\alpha \mathfrak{W}_{n-k}^{(1)} + (1 - \alpha) \mathfrak{W}_{n-k}^{(2)}) = \\ = \alpha \mathfrak{W}_k^{(1)} \mathfrak{W}_{n-k}^{(1)} + (1 - \alpha) \mathfrak{W}_k^{(2)} \mathfrak{W}_{n-k}^{(2)} \quad \text{or, if} \quad \alpha(1 - \alpha) \neq 0, \end{aligned}$$

$$(\mathfrak{W}_k^{(1)} - \mathfrak{W}_k^{(2)}) (\mathfrak{W}_{n-k}^{(1)} - \mathfrak{W}_{n-k}^{(2)}) = 0$$

from which we get the contradiction $\mathfrak{W}_m^{(1)} = \mathfrak{W}_m^{(2)}$ for all m .

3. The condition (1) is closely related to Haag's cluster property for Wightman functions²⁾.

The preceding note grew out of a discussion about a paper by E. C. G. SUDARSHAN and K. BARDAKCI³⁾.

References

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