Zeitschrift: Helvetica Physica Acta

Band: 34 (1961)

Heft: [6]: Supplementum 6. Proceedings of the International Symposium on polarization phenomena of nucleons

Artikel: Polarization phenomena in deuteron stripping reactions

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DOI: https://doi.org/10.5169/seals-513285

Polarization Phenomena in Deuteron Stripping Reactions

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Abstract. Polarization phenomena in the usual theory of (d, p) reactions (i. e., the distorted wave Butler theory without explicit spin-orbit coupling) are shown to have a common origin, which is in effect simply a measurement of the neutron transfer angular momentum direction (multiplied by quantum geometrical factors). The limitations implied by this result are discussed.

The effect of explicit spin-orbit couplings is discussed in detail for the important case where the transfer angular momentum is zero. It is shown that for this case the proton polarization is approximately given by the *derivative* of the (unpolarized) angular distribution. (Conditions are given under which this approximation is useful.)

Symmetry considerations in the distorted wave Butler theory are next discussed, and illustrated by application to Coulomb effects. Various numerical examples are cited to illustrate the qualitative predictions of the present discussion.

The great importance of the stripping process for nuclear reactions lies in the fact that from the *qualitative* features of the angular distribution one may infer, almost directly, information of use in nuclear spectroscopy. Although deuteron induced reactions had previously been considered in much experimental and theoretical detail (e.g., the Oppenheimer-Phillips process) it was the simplicity and usefulness of the Butler stripping theory that led to the widespread attention this process has received since Butler's paper [1][3]) in 1951. Since that time numerous further developments of the theory have been made, and the whole subject has been treated comprehensively in several reviews and books [2] devoted either exclusively to the stripping process, or else treating it as a special case of the so-called 'direct reaction theory'. Following Austern [2], one may divide the stripping theories into three classes:

(1) the 'crude' Butler theory, which uses plane waves [1], (2) the simple theories, which employ distorted waves in the Born approximation (with or without explicit spin-dependent interactions), and (3) the 'sophisticated' theories which attempt a more or less fundamental treatment from general reaction theory. In surveying these it seems fair to say that two

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[3]) Numbers in brackets refer to References, page 400.

points are clear: (1) The Butler theory, even in its simplest form, works surprisingly well and (2) the Butler theory, and all later modifications, are but approximations whose real applicability has yet to be fully assessed. This last point is hardly surprising since the stripping process is an instance of the general three-body rearrangement process, and above the deuteron binding energy, even the general Wigner dispersion theory becomes, in principle at least, inapplicable.

It is not our intention to attempt a discussion of the foundations of the theory; rather we shall accept the general applicability of the distorted wave Born approximation (a development that stems from HOROWITZ and MESSIAH [3]), and try to assess some of the consequences[4]). One of the bothersome difficulties of the distorted wave treatment lies in the fact that it is so general, and contains so many parameters, that it is not easy to know whether agreement (which generally is found sooner or later) is really significant or not. It is, therefore, particularly important to understand intuitively the *qualitative* predictions, and this is the real aim of the present discussion[5]).

The nuclear spectroscopic information obtained from the Butler analysis (in favorable cases) is the orbital angular momentum l (and thus the parity) of the transferred neutron[6]). Further information, such as the total angular momentum of the absorbed neutron, is desirable, and it was quickly pointed out that further measurements on the stripping process could be useful. BOYER [4] for example, suggested angular correlations with any gamma rays emitted; this gives information both for nuclear spectroscopy and the stripping reaction mechanism. It was NEWNS [5] who suggested polarization measurements, and since this conference deals with polarization we shall confine our attention to the information such measurements convey.

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[4]) There are some exceptional cases where the observed stripping pattern may be fortuitous or the result of other mechanisms (for example, $B^{10}(d, p)$), but we shall not discuss such cases. It is possible to regard the distorted wave calculations in a more subtle way, and consider that the distorted waves themselves already contain resonant effects in both the elastic and inelastic channels. This latter would then relax the stripping selection rules. For example take $Mg^{24}(d, p)Mg^{25*}$, with the Mg^{25} in the 1.611 MeV, $7/2^+$ state.

The usual selection rules require $l = 4$; but if Mg^{24} were first excited to the $l = 2$ rotational level, then $l = 2$ stripping would now be permitted. This process would, to be sure, be both improbable and not a well defined $l = 2$ stripping pattern. We shall not, however, go further into such extensions of the distorted wave calculations, since they are not directly at issue in polarization processes.

[5]) A quite different approach to the same problem (for stripping, as opposed to polarization) is the 'semi-classical' treatment of AUSTERN [2]. Although marginal in application perhaps, it nonetheless is quite helpful for an intuitive feeling for the stripping process.

[6]) The (d, p) reaction will be considered as the prototype.

In order to understand intuitively the basis for the existence of proton polarization (even in the absence of explicit proton spin orbit coupling) let us consider the (d, p) reaction from the standpoint of the stripping assumption. This we take to mean that the process may be described by the Born approximation, with the transition matrix element:

$$T_{fi} \cong \langle \psi_f^{(-)} (\boldsymbol{k}_p) \mid V_{np} \mid \psi_i^{(+)} (\boldsymbol{k}_d) \rangle . \tag{1}$$

Here $\psi_i^{(+)}$ is the initial system with (outgoing) deuteron waves, and $\psi_f^{(-)}$ is the final system with (ingoing) proton waves. For simplicity we take the $n-p$ potential, V_{np}, to be of zero range. Whether or not the integral extends over the nuclear volume is a matter of preference; the usual considerations that the stripping process occurs primarily at larger distances exclude the nucleus proper[7]). According to eq. (1), the assumption that the proton spin-orbit coupling is neglected in $\psi_f^{(-)}$ is equivalent to the assumption that the proton spin is not directly involved in the process at all, and is only affected by virtue of its coupling to form the deuteron in the initial state.

If we neglect proton spin-orbit coupling, the angular momentum relationships which govern the proton spin in the stripping process are simply:

$$\begin{aligned} &\text{a)}\ \boldsymbol{s}_p + \boldsymbol{s}_n = \boldsymbol{s}_d \\ &\text{b)}\ \boldsymbol{s}_n + \boldsymbol{l}_n = \boldsymbol{j}_n \\ &\text{c)}\ \boldsymbol{j}_i + \boldsymbol{j}_n = \boldsymbol{j}_f . \end{aligned} \tag{2}$$

That is to say, the proton spin, *by assumption*, is coupled only to the neutron spin to form the incident deuteron, and the neutron spin is coupled to its orbital angular momentum in the final system. Since, by hypothesis, the initial and final spin angular momenta ($\boldsymbol{j}_i$ and $\boldsymbol{j}_f$) are *not* observed (therefore random), one sees from (2c) that the direction of $\boldsymbol{j}_n$ is similarly *random*. One can now obtain the proton spin polarization distribution quite easily by applying semi-classical correlation techniques [6]. (This is given in detail in Appendix I.) These techniques lead at once to the relation:

$$\boldsymbol{P}_{\text{Proton}} = P_1 (\widehat{\boldsymbol{s}_p} \cdot \widehat{\boldsymbol{s}_n})\, P_1 (\widehat{\boldsymbol{s}_n} \cdot \widehat{\boldsymbol{l}_n})\, [3/l\,(l+1)]^{1/2} \langle \boldsymbol{l}_n \rangle . \tag{3}$$

In other words, the measurement of the proton spin is effectively a measurement of the direction of the neutron's orbital angular momentum, diminished

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[7]) T. HONDA and U. NAGASAKI, Proc. Phys. Soc. London, *74*, 571 (1959). These authors go further and assume that the proton is completely excluded from the nuclear region, even in the final state wave function.

by the (quantum) geometrical factors $P_1(\widehat{s_n \cdot s_p}) = 1/3$ from the deuteron spin coupling, and the factor

$$\left[\frac{3}{l(l+2)}\right]^{1/2} P_1(\widehat{s_n \cdot l_n}) = \begin{Bmatrix} (l+1)^{-1} \text{ for } j = l + 1/2 \\ (-l)^{-1} \text{ for } j = l - 1/2 \end{Bmatrix}$$

from the neutron spin coupling.

This result for the polarization is even clearer if one takes $l_n = 1$, for then the result may be written in vector notation, instead of the more complicated multipole tensor products. For a zero range np potential and $l_n = 1$, eq. (1) becomes:

$$T_{fi}(l_n = 1) \to \boldsymbol{M} \equiv \langle \psi^{(-)}_{\text{proton}}(\boldsymbol{r}) \mid \hat{\boldsymbol{r}}\, \psi_{\text{neutron}}(r) \mid \psi^{(+)}_{\text{deuteron}}(\boldsymbol{r}) \rangle. \tag{4}$$

With this, one obtains then the simple result:

$$\boldsymbol{P} = \frac{1}{3} \binom{1/2}{-1} \frac{(i)\,(\boldsymbol{M} \times \boldsymbol{M}^*)}{\boldsymbol{M} \cdot \boldsymbol{M}^*}. \tag{5}$$

From eq. (5) one can easily see all the qualitative features of the proton polarization process (under the present assumptions). Namely:

a) The maximum polarization is 33%. This is actually a result of the quantization of angular momenta, since the triangle formed by $\boldsymbol{s}_p + \boldsymbol{s}_n = \boldsymbol{s}_d$ —even though the angular momenta are 'parallel'—still has finite area, and $\widehat{\boldsymbol{s}_p} \cdot \widehat{\boldsymbol{s}_n} \neq 1$.

b) The sign of the polarization distinguishes $j = l + 1/2$ from $j = l - 1/2$ (but one must know the sign of $\boldsymbol{M} \times \boldsymbol{M}^*$ to specify either absolutely).

c) There is no polarization if the matrix element is not *complex*, and no polarization if the matrix element is a function of a *single* direction. Either one of these restrictions eliminates any polarization in Butler's original approximation where plane waves are used. (Alternatively one may say that in the plane wave approximation, the neutron is effectively taken from a plane wave along the recoil direction, and since $\boldsymbol{l}_n$ is equally probable in the plane perpendicular to this direction, $\langle \boldsymbol{l}_n \rangle = 0$.) *The importance of polarization measurements (for comparison with the theory) lies in the fact that the polarization is solely due to the distortion effects in the absence of* $\boldsymbol{l} \cdot \boldsymbol{s}$ *forces, and is therefore a sensitive test of the stripping mechanism.*

d) Since there are but two physically defined directions in the problem, $\boldsymbol{k}_p$ and $\boldsymbol{k}_d$, it is clear that $i\,\boldsymbol{M} \times \boldsymbol{M}^* = (\boldsymbol{k}_d \times \boldsymbol{k}_p)\, f(\boldsymbol{k}_p, \boldsymbol{k}_d, \boldsymbol{k}_p \cdot \boldsymbol{k}_d)$. This is the well known general result that $\boldsymbol{P}$ must lie normal to the scattering plane. (None of these conclusions is in any way affected by our specialization to $l_n = 1$.)

It is quite easy to apply the same techniques to determine the polarization of the final nucleus in the stripping reaction. The angular momentum relationships are exactly the same as before, *except that the experimental observations differ*. Thus $\boldsymbol{s}_n$ is now random (since neither $\boldsymbol{s}_p$ or $\boldsymbol{s}_d$ is observed), but $\boldsymbol{j}_n$ is not random any longer. It follows that the polarization of the final nucleus is given by:

$$\langle\widehat{\boldsymbol{j}_f}\rangle = P_1\,(\widehat{\boldsymbol{j}_f}\cdot\widehat{\boldsymbol{j}_n}) \quad P_1\,(\widehat{\boldsymbol{j}_n}\cdot\widehat{\boldsymbol{l}_n})\,\langle\widehat{\boldsymbol{l}_n}\rangle. \tag{6}$$

On using the usual definition that the polarization vector $\boldsymbol{P}$ is defined as $\langle\widehat{\boldsymbol{j}_f}\rangle/j_f$, one gets:

$$\boldsymbol{P}_{\text{nucleus}} = \left[\frac{j_f+1}{j_f(l_n)(l_n+1)}\right]^{1/2} P_1\,(\widehat{\boldsymbol{j}_f}\cdot\widehat{\boldsymbol{j}_n})\cdot P_1\,(\widehat{\boldsymbol{j}_n}\cdot\widehat{\boldsymbol{l}_n})\,\langle\widehat{\boldsymbol{l}_n}\rangle. \tag{7}$$

It will be noted at once that the neutron spin coupling introduces here quite a different result than for proton polarization. Unlike the proton case, here both $j = l \pm 1/2$ have the *same* sign. One sees further that complete polarization can result if $j_i = 0$.

As a final example of the utility of this method of deducing polarizations let us construct the deuteron polarization in the *inverse* reaction. This is, by the same general method,

$$\boldsymbol{P}_{\substack{\text{deuteron}\\ \text{(inverse)}}} = \sqrt{\frac{s_d+1}{s_d}}\;P_1\,(\widehat{\boldsymbol{s}_d}\cdot\widehat{\boldsymbol{s}_n})\,P_1\,(\widehat{\boldsymbol{s}_n}\cdot\widehat{\boldsymbol{l}_n})\,\langle\widehat{\boldsymbol{l}_n}\rangle. \tag{8}$$

However, the angular distribution of the products of a nuclear reaction induced by polarized particles and the polarization of the particles produced in the inverse reaction are related [7] under very general conditions by the equation:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{pol.}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol.}}\left[1 - \frac{3\,j}{j+1}\,\boldsymbol{P}^I\cdot\boldsymbol{P}^{II}\right]. \tag{9}$$

(Here $\boldsymbol{P}^I$ and $\boldsymbol{P}^{II}$ are the polarization vectors of a particle of spin j, with I referring to the particle initiating the reaction and II to the inverse reaction.) We may now substitute eq. (8) in eq. (9) and, if we further recall eq. (3), then one obtains

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{pol}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol.}}\left[1 + 3\,\boldsymbol{P}_{\text{deuteron}}\cdot\boldsymbol{P}_{\text{proton}}\right]. \tag{10}$$

This relation was first obtained by SATCHLER[8]) and is of interest experimentally in that reactions produced by polarized deuterions may be a

[8]) G. R. SATCHLER, Nucl. Phys. *6*, 543 (1958). Note that the sign convention in eq. (10) differs from that given in this reference.

100% effect, as opposed to the 33% limit of the proton polarization. One further notes that *tensor* moments do not enter this relation (eq. (10)). Referring to the angular momentum coupling diagram one sees that this is due *solely to the fact that the angular information on the deuteron spin is coupled through the neutron spin.* Thus $P_\nu(\widehat{s_d} \cdot \widehat{s_n})$ enters for the tensor moments, but (because $s_n = 1/2$) ν must be 0 or 1 only.

Finally one may ask as to the generality of the results given by eqs. (3), (7) and (9). It is clear that they are all a direct consequence of the assumption that the proton spin is not directly coupled to the reaction in the stripping approximation without spin-orbit forces, and are no longer valid upon considering spin-orbit couplings[9]). This is important because several examples are already known where $P_p > 1/3$, and hence the Butler theory is not always applicable.

To summarize: *Polarization in the stripping approximation, without explicit spin-orbit coupling on the proton, is essentially a measurement of the transfer angular momentum direction, multiplied by the appropriate quantum geometrical factors.*

Let us turn next to the effects of spin-orbit coupling. The general results that can be obtained using the stripping hypothesis, are quite involved because of the complicated nature of the angular momentum couplings. This general result is given in Appendix II, but will not be discussed otherwise[10]). Not many numerical calculations have been carried out with spin-orbit coupling included, but in these [8] the effect is found to be small. (Since $P_{\text{proton}} \leqq 1/3$ is not valid here, there can in principle be quite large effects.)

The special case where $l_n = 0$ is of particular interest, because here it is clear that proton spin-orbit effects are the only cause of polarization. In order to obtain most conveniently the desired polarization formula, it is useful to recall that for spin 1/2 particles and photons the polarization can be *formally*[11]) expressed quite simply in terms of the angular distribution [9]. Thus if the angular distribution is given by:

$$\left(\frac{d\sigma}{d\Omega}\right) = \sum_{\nu\, l\, j\, l'\, j'} B_\nu\,(l\,j\,l'\,j')\; P_\nu\,(\cos\theta)\;,$$

[9]) Dr. J. GAMMEL has also arrived at similar conclusions in a paper appearing in these Proceedings. The authors are indebted to Dr. GAMMEL for discussions on these topics prior to delivery of this paper.

[10]) See also the paper by GOLDFARB, these Proceedings.

[11]) It is essential to call attention to the formal nature of this result, for the polarization and the angular distribution are in principle independent functions and one cannot be obtained from the other. The difficulty resides in the fact that in eq. (11) only the symmetric (real) part of the dynamical factors in B enters, while in eq. (12) only the anti-symmetric (imaginary) part enters; hence eq. (11) and eq. (12) only formally involve the 'same' dynamical factors.

$$\text{with } B_\nu\,(l\,j\,l'\,j')^* = B_\nu\,(l'\,j'\,l\,j)\,, \tag{11}$$

then:

$$\boldsymbol{P}(\theta) = \widehat{\boldsymbol{n}} \left(\frac{d\sigma}{d\Omega}\right)^{-1} (i) \sum_{\nu\,l\,j\,l'\,j'} B_\nu\,(l\,j\,l'\,j')\,(-)^{\nu+l'+j'+1/2}\,.$$

$$\cdot \left[\frac{6\,(2\nu+1)}{\nu+1}\right]^{1/2} C^{l\,l'\,\nu}_{00}\, X \begin{pmatrix} l & j & 1/2 \\ l' & j' & 1/2 \\ \nu & \nu & 1 \end{pmatrix} .\, W \left(l\,j\,l'\,j';\, \frac{1}{2}\,\nu\right)^{-1} \cdot \tag{12}$$

$$\cdot\, P^{(0)}_\nu\,(\cos\theta)\,.$$

In eq. (12) the unit vector $\widehat{\boldsymbol{n}}$ has the direction $\boldsymbol{k}_d \times \boldsymbol{k}_p$, in accord with the 'Basel convention'. An identity [10] for the X coeffizient that appears in eq. (12) is given in Appendix III; this simplifies eq. (12) but is less useful for the usual manipulations of the Racah algebra. For the special case that $l_n = 0$, one finds the result[12]) that the angular distribution is:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\lambda^2{}_d}{2\,s_d+1}\right) \left(\frac{2\,j_f+1}{2\,j_i+1}\right) \sum_\nu B_\nu\, P_\nu\,(\widehat{\boldsymbol{k}}_p \cdot \widehat{\boldsymbol{k}}_d)\,, \tag{13}$$

where:

$$B = \frac{1}{4} \sum \bar{Z}\,(l_p\,j_p\,l_p'\,j_p';\; s_p\,\nu)\,(-)^{\,j_p - s_p}$$

$$\bar{Z}\,(l_d\,j_d\,l_d'\,j_d';\; s_d\,\nu)\,(-)^{\,j_d - s_d}\,.$$

$$\cdot\,(-)^\nu\,[(2\,j_p+1)\,(2\,j_p'+1)\,(2\,j_d+1)\,(2\,j_d'+1)]^{1/2}\, W\,(j_p'\,\nu\,j_n\,j_d;\, j_p\,j_d')$$

$$A^{\,l_n\,j_n}_{\,l_p\,j_p;\,l_d\,j_d} \qquad A^{*\,l_n'\,j_n}_{\,l_p'\,j_p';\,l_d\,j_d}\,, \tag{14}$$

with,

$$A^{\,l_n\,j_n}_{\,l_p\,j_p;\,l_d\,j_d} \equiv e^{i\,[\delta(l_p\,j_p)\,+\,\delta(l_d\,j_d)]}\, C^{\,l_d\,l_n\,l_p}_{\,0\;\;0\;\;0}\,.$$

$$\cdot \sqrt{(2\,l_n+1)\,(2\,s_n+1)}\, X \begin{pmatrix} l_d & s_d & j_d \\ l_n & s_n & j_n \\ l_p & s_p & j_p \end{pmatrix} \cdot G^{\,l_n\,j_n}_{\,l_p\,j_p;\,l_d\,j_d}\,. \tag{15}$$

$G^{\,l_n\,j_n}_{\,l_p\,j_p;\,l_d\,j_d}$ is discussed following eq. II-3 of the Appendix.

It is convenient now to neglect the spin-orbit coupling on the deuteron, and then:

$$\left(\frac{d\sigma}{d\Omega}\right)_{l_n=0} = \frac{\lambda^2{}_d}{2\,s_d\,1} \cdot \left(\frac{2\,j_f+1}{2\,j_i+1}\right) \sum_\nu B_\nu\,(l_n=0)\, P_\nu\,(\widehat{\boldsymbol{k}}_p \cdot \widehat{\boldsymbol{k}}_d)\,,$$

$$B_\nu\,(l_n=0) = \left(\bar{Z}\left(l\,j\,l\,j;\,\frac{1}{2}\,\nu\right)\right)^2 (A_{lj})\,(A^*{}_{lj})\,,$$

[12]) Here, and elsewhere unless stated otherwise, we are assuming the stripping hypothesis with a zero-range np potential. The further approximation is made that the center of mass is fixed throughout on the target nucleus.

$$A_{lj} \equiv exp\ i(\delta_{lj}^{(p)} + \delta_l^{(d)})\ G_{lj;\,l}^{0,\,1/2}. \tag{16}$$

It has been noted many times in the past that the polarization obtained in calculations with the optical model are qualitatively similar to a derivative of the angular distribution. Recently RODBERG [11] has shown that this 'derivative relation' is in fact semi-quantitative for small spin-orbit coupling, and that moreover the ratio

$$|\boldsymbol{P}|\left(\frac{d\sigma}{d\Omega}\right)\Big/\left[\frac{d}{d\theta}\left(\frac{d\sigma}{d\Omega}\right)\right]$$

is a measure of the strength of the spin orbit potential. RODBERG's physical argument is more general, it appears, than his specific application (to the optical model), and it is tempting to apply it to the stripping process as well. For the situation described by eq. (16) it is shown in Appendix IV that for small spin-orbit forces on the proton, the polarization*) is

$$\boldsymbol{P} \cong \hat{\boldsymbol{n}} \left(\frac{d\sigma}{d\Omega}\right)^{-1} \frac{d}{d\theta}\left(\frac{d\sigma}{d\Omega}\right) \cdot \frac{1}{2}\langle\beta_l\rangle\,, \tag{17}$$

$$\text{where } \langle\beta_l\rangle = \langle\left(\frac{-2}{k}\right) e^{-2i\,\delta_l}\int_0^\infty dr\ U_l^{(-)*}(r)\ V_{\substack{\text{spin}\\\text{orbit}}}\ U_l^{(+)}(r)\rangle.$$

(If one assumes that the deuteron spin-orbit potential is small, then the same result applies at once to the more general case given by eq. (15). Although we do *not* have a proof, it is quite plausible that eq. (17) is approximately correct *for the spin-orbit contribution to* $\boldsymbol{P}$ in the general case. One sees that this is qualitatively correct from examining the results of ref. (15), but the published data do not permit much more than a very rough estimate.)

The origin of the factor of 1/2 in eq. (17) (as opposed to the result of ref. (11) for elastic scattering) comes from the fact that the spin-orbit potential acts on the outgoing wave only.

Qualitatively then one may say that for stripping reactions with $l_n = 0$, a measurement of $\boldsymbol{P}$ is approximately the same thing as measuring the proton polarization in the elastic scattering from the final nucleus.

It was early noticed in the numerical calculations involving BUTLER's theory by the use of distorted waves [3, 12] that Coulomb effects and nuclear scattering effects tended to compensate, thus improving the range of applicability of the simpler theory (but no general basis for the cancellation [other than the difference in sign of these potentials] has been given). NEWNS and REFAI [8] gave a qualitative argument to show

*) *Note added in proof:* Recent numerical calculations of Mr. WILLIAM GIBBS for $C^{12}(d, p)C^{13*}$ ($l_n = 0$ case with spin-orbit coupling), indicate good agreement with this "derivative relation", particularly for giving the minima in the polarization.

that absorptive distortion effects on the incident particle tended to cancel absorptive distortion effects on the emergent particle in producing polarization. Recently OKAI [13] and SATCHLER [14] have investigated the origin of this cancellation. It is shown in [14] that if the two conditions: 1) $|\boldsymbol{k}_p| = |\boldsymbol{k}_d|$, and 2) $m_p M_f V_{pf} = m_d M_i V_{di}$, hold, then the polarization is *exactly* zero. To understand why this occurs one need only note that the two conditions given are equivalent to eliminating *any* physical distinction—except direction of motion—between the incident and emergent particles, since both have equivalent length scales and equivalent interactions (i.e. same dimensionless strength parameter, and same length scale).

There is, however, now a physical operation which can interchange the role of the two wave functions in the matrix element by interchanging the two directions $\hat{\boldsymbol{k}}_p$ and $\hat{\boldsymbol{k}}_d$. This exchange may be accomplished in two steps: 1) a rotation by π about a direction bisecting $\theta = \cos^{-1} \hat{\boldsymbol{k}}_p \cdot \hat{\boldsymbol{k}}_d$, and 2) a reversal in the directions $\hat{\boldsymbol{k}} \to -\hat{\boldsymbol{k}}$. These two operations give an invariance operation for the process. But $\boldsymbol{P}$ reflects under the same process; therefore, $\boldsymbol{P}$ must vanish. Clearly the vanishing of $\boldsymbol{P}$ implies that distortions (whether absorptive or not) must give cancelling effects on the polarization. The second condition (above) requires that the deuteron potential, V_{di}, must be about *half as strong* (since $m_p M_f \cong 1/2\, m_d M_i$) as that for the proton; since it is usual to take the two potentials to be about of the same strength this supports the general statement that deuteron distortion is the dominant effect in stripping.

For Coulomb effects on the stripping one sees that the second condition is *never* satisfied since this would require $k\,\eta$ to be the same, whereas $k_d \eta_d = 2\, k_p \eta_p$. (It is interesting to note that for the Coulomb field, these two conditions are necessary for the classical limit to exist, as in, for example, Coulomb excitation.)

For $l_n = 0$, stripping calculations using Coulomb distorted 'plane' waves lead to an integral which (for $R = 0$) can be expressed in closed form, and this result can be exploited to give a qualitative understanding of the effects of large Coulomb fields [15]. Unfortunately no similar result can be obtained for $l_n \geqq 1$. The best that can be done (without employing series) is the integral for $\boldsymbol{M}$ given in Appendix V. From this one sees that, *for the special case* $|\boldsymbol{k}_p| = |\boldsymbol{k}_d|$, the effect of the Coulomb field is to introduce besides the recoil vector $(\boldsymbol{k}_d - \boldsymbol{k}_p)$ a second independent vector: $(\eta_p \boldsymbol{k}_d - \eta_d \boldsymbol{k}_p)$ which leads to the result that

$$\boldsymbol{P} = (\eta_d - \eta_p)\,(\boldsymbol{k}_d \times \boldsymbol{k}_p)\, f\,(k_p, k_d, \hat{\boldsymbol{k}}_p \cdot \hat{\boldsymbol{k}}_d)\,.$$

This explicitly shows the cancellation of Coulomb distortion effects on the polarization. Moreover, since $k_d \simeq k_p$ implies that $\eta_d = 2\,\eta_p$ it is clear that the deuteron distortion dominates once again.

With these qualitative results in mind let us look now at some of the numerical results[13]).

Example 1: $B^{10}(d, p)B^{11}$, $Q = 9.24$ MeV, $l = 1$, $j = 3/2$.

The data for this example are taken from the summary of HENSEL and PARKINSON [16] and give the angular distribution of protons for an incident deuteron energy of 8.1 MeV (lab.). The Coulomb parameters are: $\eta_d \simeq 0.4$, $\eta_p \simeq 0.2$; hence the wave numbers are approximately equal: $k_p \simeq k_d$. To carry out a distorted wave calculation the ancillary task of an optical model fit to the elastic deuteron scattering (from B^{10}) and the elastic proton scattering (from B^{11}) must be carried out. These fits (compared to an experiment at approximately the desired energy) are shown in figures 1 and 2. The optical model data are given in table I.

Table I

Parameters characterizing the three examples illustrated

	$B^{10}(d, p)B^{11}$ *a*	$B^{10}(d, p)B^{11}$ *b*	$Ti^{48}(d, p)Ti^{49*}$ *c*	$O^{16}(d, p)O^{17}$ *d*
E_d(MeV)	8.1	8.1	2.6	19.0
Q(MeV)	9.24	9.24	4.46	1.918
l	1	1	1	2
j	3/2	3/2	3/2	5/2
R(f.)	5.41	6.15	6.18	3.944
Wood-Saxon well parameters:				
V_{id}(MeV)	−60	−50	−44	−40
W_{id}(MeV)	−17	−14	−13	−15
a_{id}(f.)	0.70	0.68	0.7	0.75
R_{id}(f.)	3.66	3.23	5.32	3.8
V_{fp}(MeV)	−50	−50	−60	−38
W_{fp}(MeV)	−11	−8	−7	−10
a_{fp}(f.)	0.4	0.4	0.45	0.5
R_{fp}(f.)	2.9	2.9	4.36	3.35

Turning now to the Butler curve (plane waves) first, figure 3, one sees that the shape of the peak is rather well fitted but is quite poor beyond

[13]) Examples 1 and 2 are taken from the paper of W. TOBOCMAN, Phys. Rev. *115*, 98 (1959). Example 3 is preliminary, and is taken from the thesis of Mr. WILLIAM GIBBS, Rice Institute (1960), (in preparation). We are indebted to both Dr. TOBOCMAN and Mr. GIBBS for permission to include their examples. (It should be noted that the sign convention used in the paper cited is not explicitly stated, but apparently the $\boldsymbol{k}_d \times \boldsymbol{k}_p$ convention is used, judging from the quoted experimental data.)

50°. From the high energy and low Z value one expects the simplest Butler theory to be reasonably applicable although, as suggested by WILKINSON, the relatively high Q value may be a factor opposed to this conclusion.

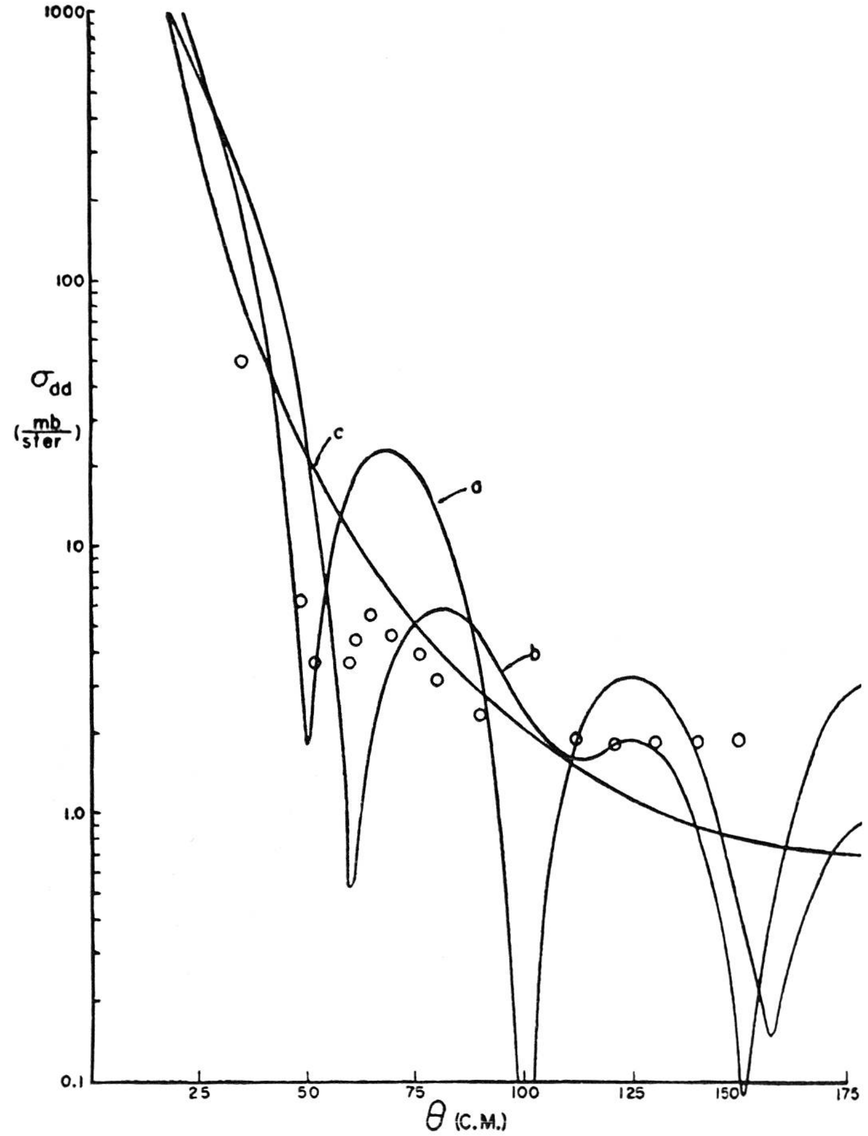

Figure 1

Cross section for the elastic scattering of 8.1 MeV (lab) deuterons on B^{10}. Curves a and b are the cross sections predicted by the potentials a and b given in table I. Curve c is the Rutherford cross section. (The experimental data (circles) are from ref. 16 and represent the elastic scattering of 7.7 MeV (lab) deuterons on Be^9.)

Introducing now Coulomb effects only we obtain figure 4. Because $\eta_d > \eta_p$; $k_p \cong k_d$; and $j = l + 1/2$ we expect the polarization to be initially positive in the convention adopted. This is borne out by the numerical results. Note that $P = 0$ at almost precisely the point where the Butler curve has its first zero.

With the optical potentials as well as Coulomb effects, but cutting off the integral at R, we get figure 5. Note that the Coulomb term dominates the (opposing) optical term in the forward direction (as it must) but that otherwise little else can be said about the result in general.

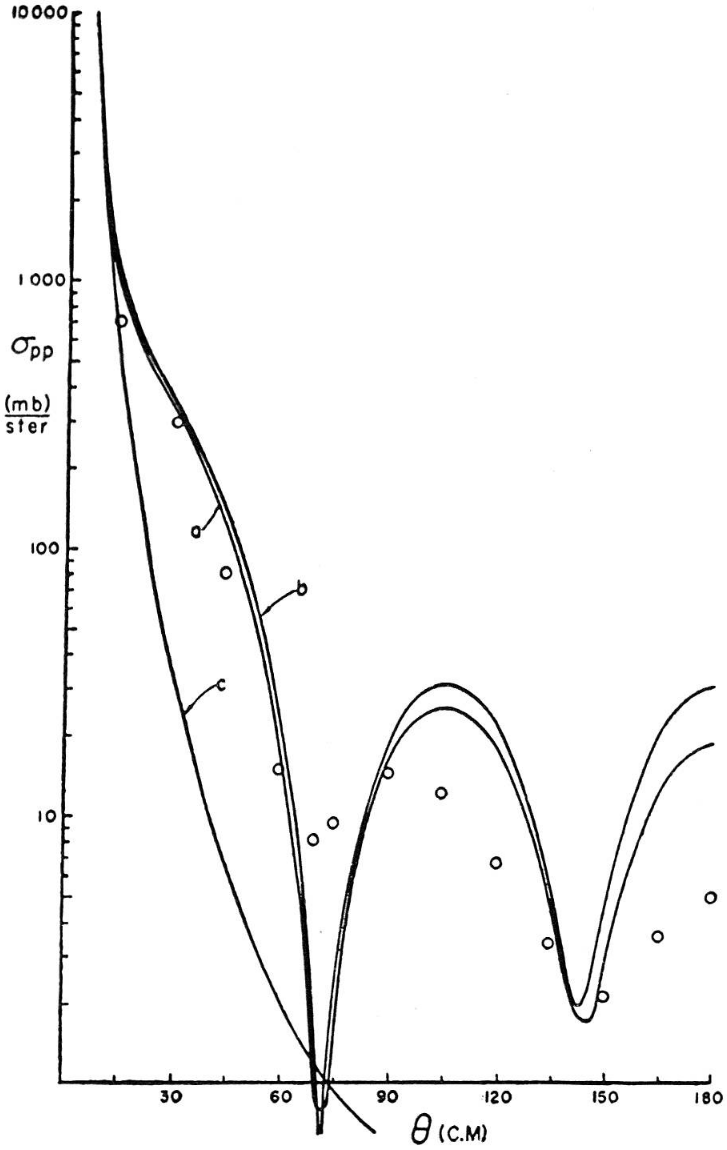

Figure 2

Cross-section for the elastic scattering of 17.44 MeV (lab) protons on B^{11}. Curves *a* and *b* are calculated with potentials *a* and *b* given in table I. (The circles represent the experimental elastic cross section for 17.0 MeV (lab) protons on B^{10}.)

Finally without cut-off we get figures 6 and 7 for two choices of the deuteron optical potential. The significant points here are 1) the extreme sensitivity of the polarization in the two cases and 2) the qualitative explanation in terms of cancellation which these potentials suggest. Taking MVR^2 as the appropriate parameter, in case (a) the deuteron to proton parameter is about 4 to 1 and in case (b) about 4 to 3. Thus the latter case should show *less* polarization, which is observed.

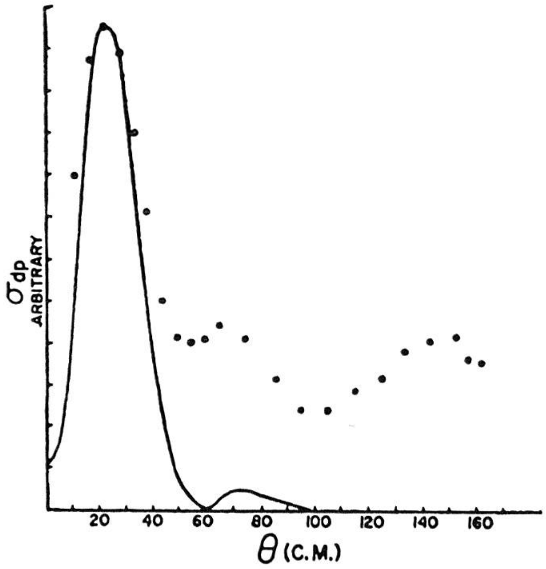

Figure 3

$B^{10}(d, p)B^{11}$ cross section according to the Butler theory for 8.1 MeV (lab) deuterons taking $l = 1$ and $j = 3/2$. (The experimental points are arbitrarily normalized to fit the calculated curve.)

Example 2: $Ti^{48}(d, p)Ti^{49*}$, $Q = 4.46$ MeV, $l = 1$, $j = 3/2$, $R = 6.18\,f$.

The angular distribution for 2.6 MeV incident deuterons is taken from Pratt [17]. Figure 8 shows the Butler plane wave curve: this is qualitatively inconsistent with the data. Since this case is chosen to be both low energy and high Z (the Coulomb parameters are $\eta_d \cong 3$, $\eta_p \cong 1.25$), Coulomb effects should be large, and the lack of agreement with a plane wave calculation is to be expected. In figure 9 only the Coulomb effects are introduced, and the fit is improved considerably. Since $k_p \cong k_d$ and $\eta_d > \eta_p$ with $j = l + 1/2$ the polarization should initially be positive as is obtained. Finally in figure 10 the non-cut-off distorted wave theory results are shown, and the agreement seems quite good.

The problem as to whether or not to extend the integrals over the nuclear interior is not clear-cut. In figure 11 we show the effect of various neutron wave functions on the polarization and angular distribution; in figure 12 the corresponding wave functions are shown. The great sensitivity of the results to the interior is disturbing, for stripping is best justifiable as a long range effect.

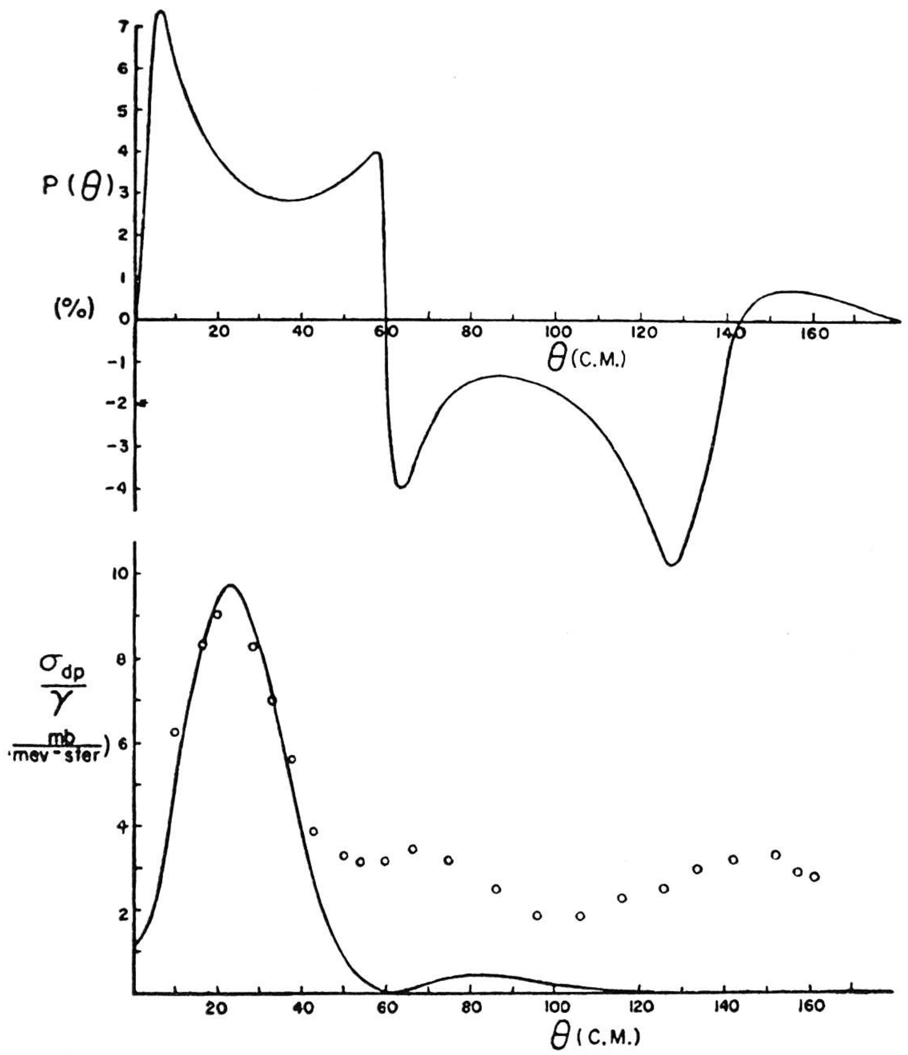

Figure 4

$B^{10}(d, p)B^{11}$ cross section and polarization using the Butler theory modified to include Coulomb effects, with $l = 1$ and $j = 3/2$, 8.1 MeV deuterons.

In this connection it might be well to call attention to an unusual feature of the direct reaction process which has not, to the authors's knowledge, been explicitly discussed[14]). Consider the case where the process is at such low energies that the reaction is dominated completely by Coulomb forces. From the asymptotic form of the Coulomb matrix element (as given in Appendix V say, since l is not important in this limit) one can obtain the asymptotic form of the direct reaction cross section. Now for a resonance process, this asymptotic form is simply the product of the two penetration factors, i. e. $\sim e^{-2\pi\,\eta_d} \cdot e^{-2\pi\,\eta_p}$. Comparing this to the direct reaction result one may define an enhancement factor (E. F.), i. e., E. F. = Ratio of direct process to resonance process (compound nucleus). One finds that for $E_d \rightarrow 0$, E. F. $\sim \exp\{\eta f(x)\}$, where $\eta \equiv 4\,Ze^2\,M/\hbar^2\alpha$ (with α being the reciprocal radius of the

[14]) It is implicitly contained, however, in the work of LANDAU and LIFSHITS, J. Exptl. Theor. Phys. *18*, 750 (1948); and in the papers of ref. [15].

deuteron (4.32 f^{-1})) and $x \equiv k_p/\alpha$. The function $f(x)$ is: $f(x) \equiv [(2+x^2)^{1/2}-1]^{-1} + x^{-1}\tan^{-1}(x/[2-(2+x^2)^{1/2}])$. For $Q \to 0$, E. F. $\to \exp(2^{1/2}(1+2^{-1/2})^2\eta)$; for $Q \to \infty$, the enhancement factor becomes $\exp\{\eta_p(4+3\pi)\}$, which approaches 1, as it must. Clearly then, the direct process is considerably enhanced by processes taking place at very large distances, i. e. the deuteron does *not* have to penetrate the barrier to react.

The most striking instance of this occurs for the case where $k_1\eta_1 = k_2\eta_2$ (unlike the (d, p) case). For this case the direct reaction goes as $e^{-2\pi|\eta_1-\eta_2|}$; in other words for $\eta_1 = \eta_2$ the barrier to the *ingoing particle cancels the barrier for the outgoing particle*. Since barriers oppose for either direction this is a surprising, but correct, result[15]).

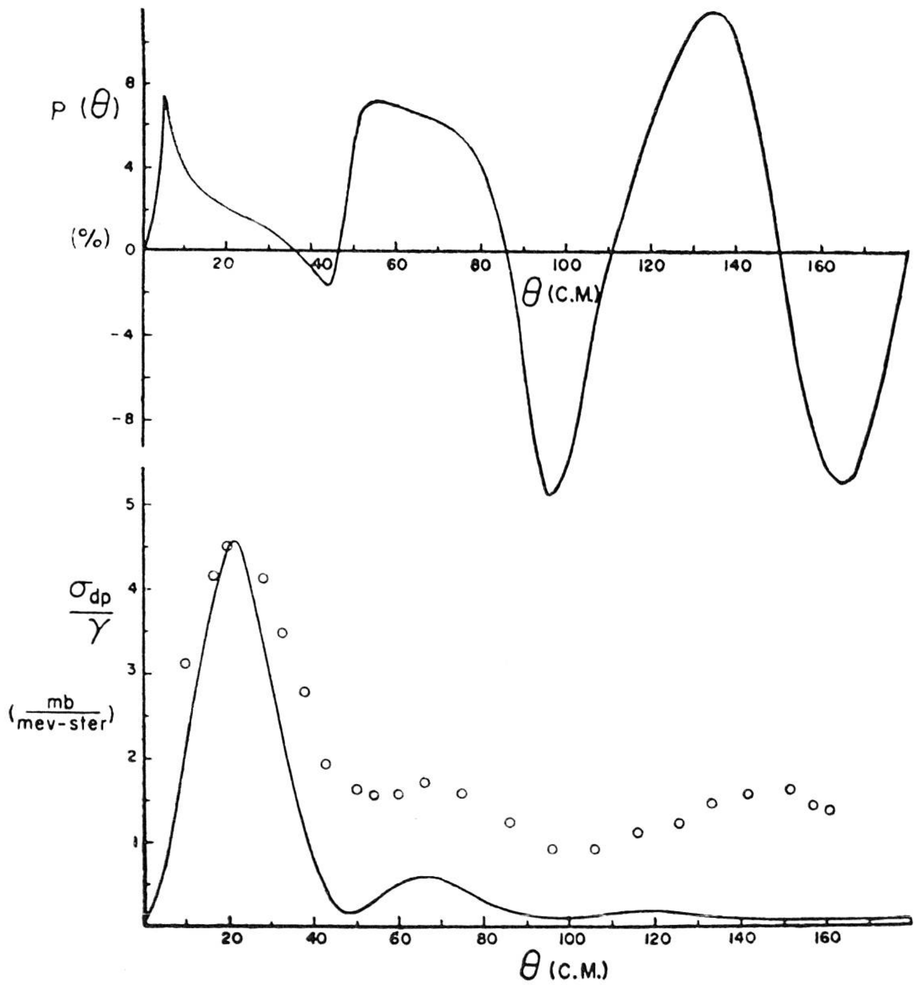

Figure 5

$B^{10}(d, p)B^{11}$ cross section and polarization using the distorted wave modification of the Butler theory, with cut-off at R. The potentials given in case a of table I were used in the calculation.

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[15]) This effect is well-known in Coulomb excitation, and might be important in, say, Coulomb induced fission. Data in the barrier inhibited region (for protons on U^{238}) do not decide for or against such a process. (Unpublished calculations 1956, of R. M. THALER and L. C. BIEDENHARN.)

Example 3: $O^{16}(d, p)^{17}O$, $Q = 1.918$ MeV, $R = 3.944$ *f.*, $l = 2$, $j = 5/2$.

The deuteron energy in this example is 19.0 MeV, and the Coulomb parameters are $\eta_d \simeq 0.4$ and $\eta_p \simeq 0.25$. Coulomb effects should be quite small. The required fits to the elastic scattering for the deuteron and the

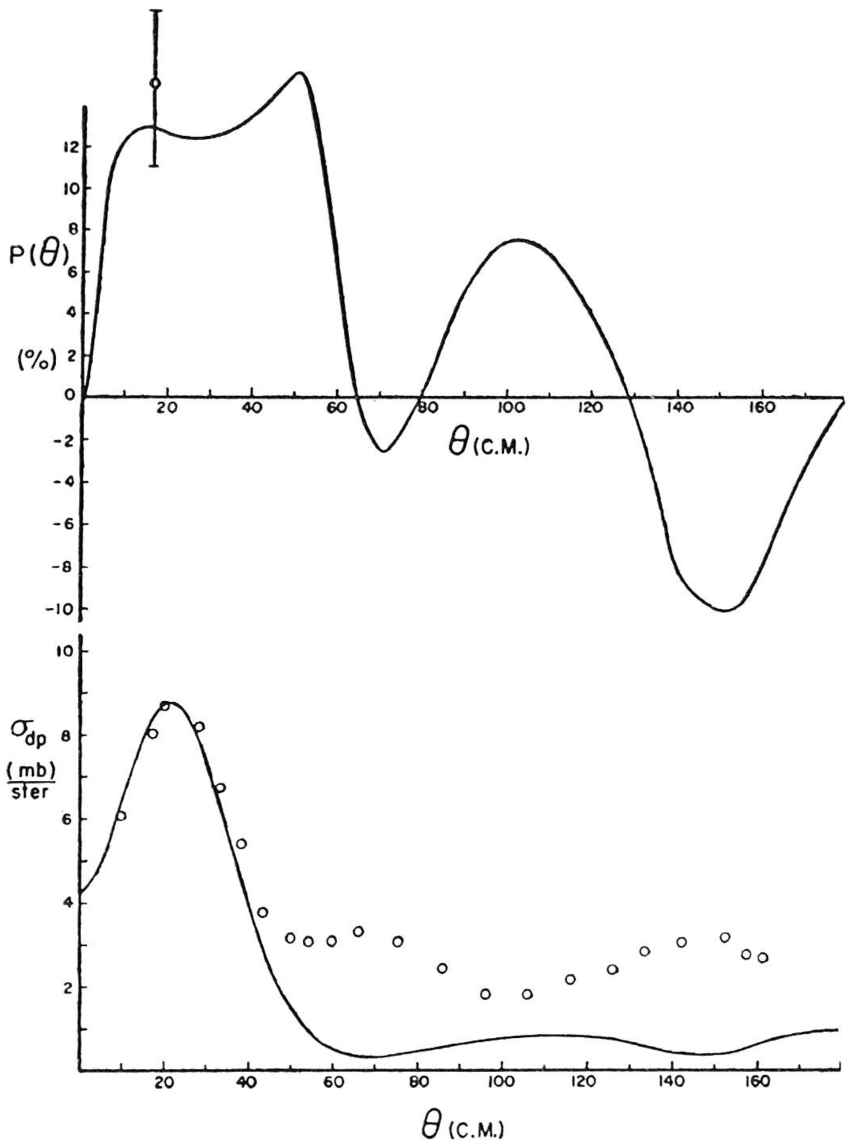

Figure 6

$B^{10}(d, p)B^{11}$ cross section and polarization using the distorted wave modification of the Butler theory without cut-off. The optical potentials are those of case *a*, table I.

proton are shown in figure 13. The data for the potentials are that for case *d* in table I. The results obtained for the complete distorted wave treatment, with and without cut-off, are shown in figure 14.

The small η approximation discussed in Appendix V ought to be qualitatively applicable to this example (even though the l value is 2). From the results in the appendix one sees that the initial (small θ) polarization due to Coulomb effects can be *negative* for $k_p^2 > k_d^2$ and small k_n^2, such as obtains in this example. One notes in accord with this that both the cut-off and non-cut-off cases show negative polarization near the forward direction. Note also that in both cases the polarization vanishes almost exactly at the first extremum in the angular distribution.

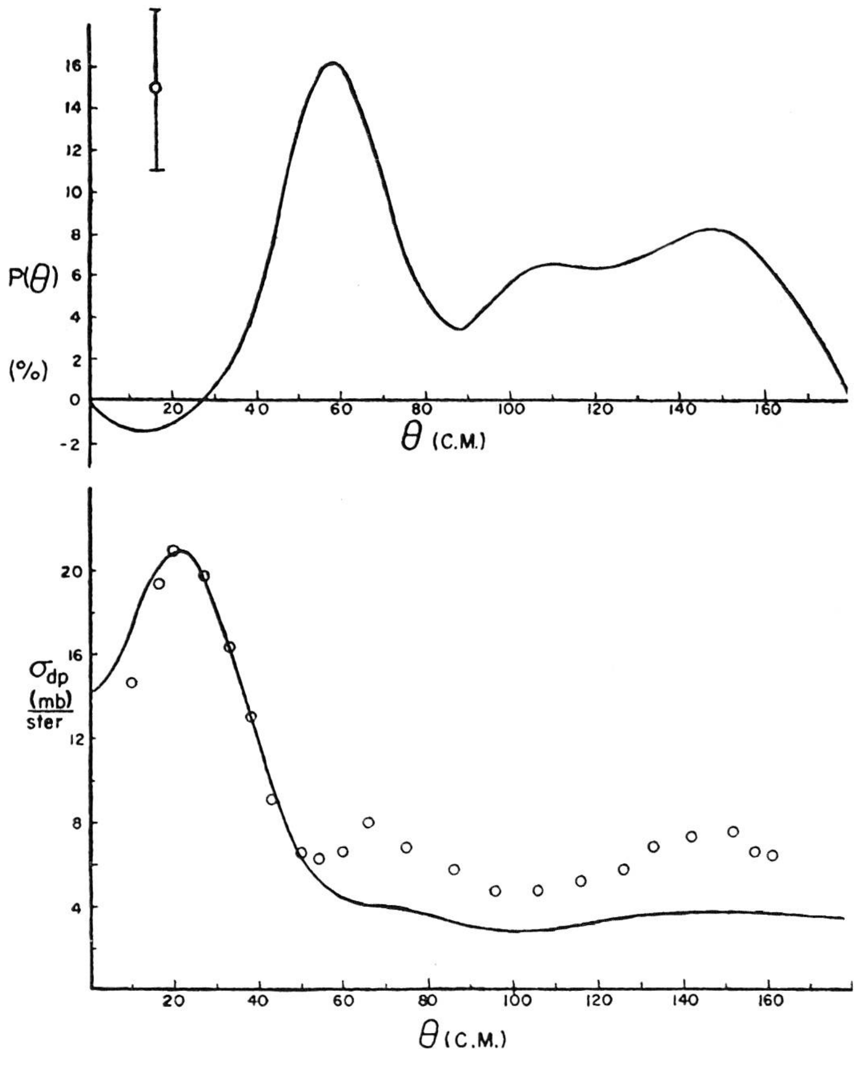

Figure 7

$B^{10}(d, p)B^{11}$ cross-section and polarization using the distorted wave modification of the Butler theory without cut-off. The optical potentials are those of case *b*, table I.

Unlike example 2, where the cut-off greatly varied P but not $(d\sigma/d\Omega)$, this case shows just the opposite behaviour!

To conclude then, it is clear that anything more then a qualitative understanding of the stripping process is a very arduous task. It is equally clear that there is quite a lot of work to be done in this field—experimental, numerical, and theoretical.

Acknowledgements: In the preparation of this paper the authors were greatly helped by many colleagues, in particular Drs. GOLDFARB, IVASH, and TOBOCMAN, and by Messrs. WILLIAM GIBBS and THOMAS GRIFFY.

Appendix I

In the distorted wave Born approximation for stripping reactions the proton polarization (assuming no proton spin orbit coupling) is governed by the angular momentum relations:

$$\boldsymbol{s}_n + \boldsymbol{s}_p = \boldsymbol{s}_d \qquad (I-1)$$
$$\boldsymbol{s}_n + \boldsymbol{l}_n = \boldsymbol{j}_n\,,$$

with the directions of neither $\boldsymbol{s}_d$ or $\boldsymbol{j}_n$ being observed.

Using (classical) vectors to represent the angular momenta, one can diagram these relations as shown in figure (I-1).

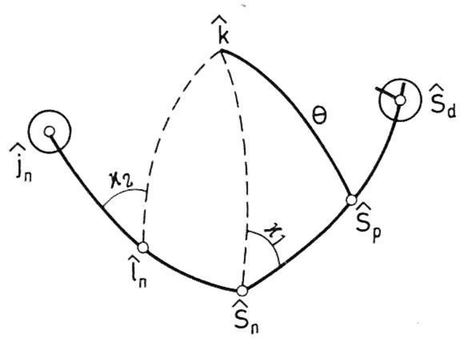

Figure (I–1)

The (classical) angular momentum vectors are represented by their intersection with the surface of a unit sphere with all vectors taken to issue from a common origin. Since $\boldsymbol{s}_d = \boldsymbol{s}_n + \boldsymbol{s}_p$, and $\boldsymbol{s}_n + \boldsymbol{l}_n = \boldsymbol{j}_n$ the representing points for these cases lie along arcs of great circles. The vectors $\boldsymbol{j}_n$ and $\boldsymbol{s}_d$ are random (unobserved), and thus the angles χ_1 and χ_2 are random. $\boldsymbol{k}$ represents an arbitrary direction in space.

Taking an arbitrary axis in space, $\widehat{\boldsymbol{k}}$, the probability that $\boldsymbol{s}_p$ make an angle (θ, ϕ) with $\widehat{\boldsymbol{k}}$ must be independent of ϕ (cylindrical symmetry), and we, therefore, represent this probability by $W(\theta)$ and develop it as a Legendre series.

$$W(\theta) = \sum_\nu \frac{2\,\nu+1}{2} \langle P_\nu\,(\widehat{\boldsymbol{s}_p} \cdot \widehat{\boldsymbol{k}})\rangle\, P_\nu\,(\widehat{\boldsymbol{k}} \cdot \widehat{\boldsymbol{s}_p})\,. \qquad (I-2)$$

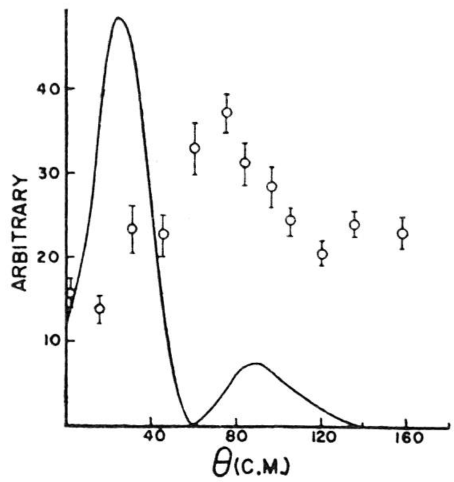

Figure 8

$Ti^{48}(d, p)Ti^{49*}$ cross section for 2.6 MeV (lab) deuterons, calculated for the Butler theory with $l = 1$, $j = 3/2$. (The experimental points are arbitrarily normalized.)

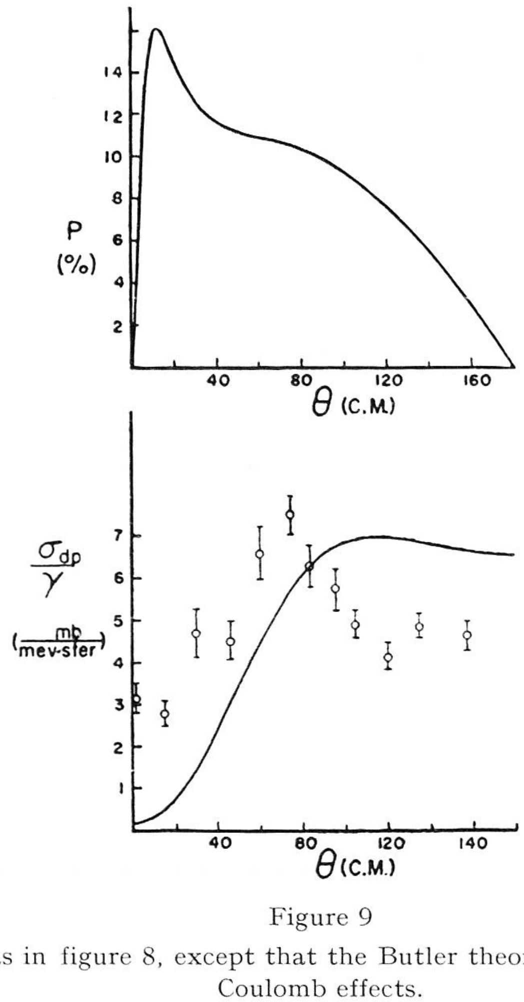

Figure 9

$Ti^{48}(d, p)Ti^{49*}$ as in figure 8, except that the Butler theory is modified to include Coulomb effects.

Here $\langle P_\nu \rangle$ represents an average over the random directions of $\boldsymbol{j}_n$ and $\boldsymbol{s}_d$. Using the addition theorem to carry out the averages (over χ_1 and χ_2) one finds successively:

$$\begin{aligned}\langle P_\nu (\widehat{\boldsymbol{s}_p} \cdot \widehat{\boldsymbol{k}}) \rangle &= P_\nu (\widehat{\boldsymbol{s}_p} \cdot \widehat{\boldsymbol{s}_n}) \langle P (\widehat{\boldsymbol{s}_n} \cdot \widehat{\boldsymbol{k}}) \rangle \\ &= P_\nu (\widehat{\boldsymbol{s}_p} \cdot \widehat{\boldsymbol{s}_n}) \quad P_\nu (\widehat{\boldsymbol{s}_n} \cdot \widehat{\boldsymbol{l}_n}) \langle P_\nu (\widehat{\boldsymbol{l}_n} \cdot \widehat{\boldsymbol{k}}) \rangle, \end{aligned} \qquad \text{(I–3)}$$

and hence:

$$W(\theta) = \sum_\nu \left(\frac{2\,\nu + 1}{2} \right) P_\nu (\widehat{\boldsymbol{s}_p} \cdot \widehat{\boldsymbol{s}_n}) \, P_\nu (\widehat{\boldsymbol{s}_n} \cdot \widehat{\boldsymbol{l}_n}) \, \langle P_\nu (\widehat{\boldsymbol{l}_n} \cdot \widehat{\boldsymbol{k}}) \rangle \, P_\nu (\widehat{\boldsymbol{s}_p} \cdot \widehat{\boldsymbol{k}}) \, . \qquad \text{(I–4)}$$

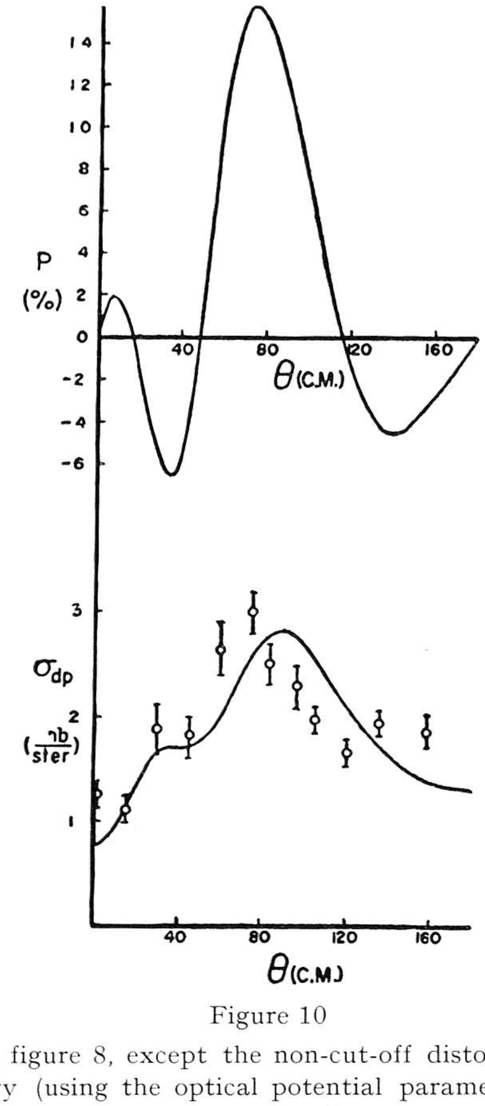

Figure 10

$Ti^{48}(d, p)Ti^{49*}$ as in figure 8, except the non-cut-off distorted wave modification of the Butler theory (using the optical potential parameters of case *c*, table I) is employed.

From (I-4) it follows that:

$$\langle P_1 (\widehat{\boldsymbol{s}_p} \cdot \widehat{\boldsymbol{k}}) \rangle = \langle \widehat{\boldsymbol{s}_p} \cdot \widehat{\boldsymbol{k}} \rangle = P_1 (\widehat{\boldsymbol{s}_p} \cdot \widehat{\boldsymbol{s}_n}) \, P_1 (\widehat{\boldsymbol{s}_n} \cdot \widehat{\boldsymbol{l}_n}) \, \langle \widehat{\boldsymbol{l}_n} \cdot \widehat{\boldsymbol{k}} \rangle,$$

or, since $\boldsymbol{k}$ is arbitrary,

$$\langle \widehat{s_p} \rangle = P_1 (\widehat{s_p} \cdot \widehat{s_n}) \, P_1 (\widehat{s_n} \cdot \widehat{l_n}) \quad \langle \widehat{l_n} \rangle. \tag{I–5}$$

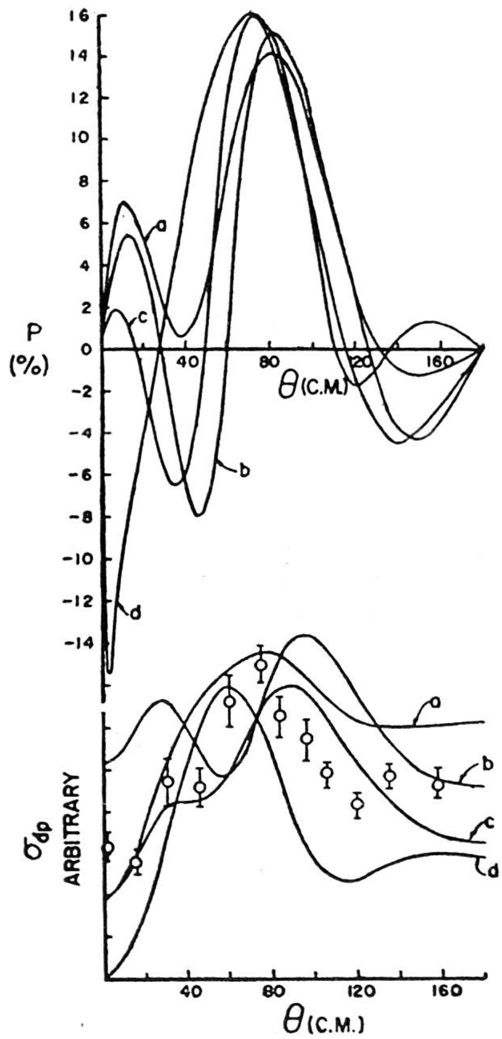

Figure 11

$Ti^{48}(d, p)Ti^{49*}$ as in figure 10, using various choices of the captured neutron wave function. (Note that various scale factors are used in the cross-section curves.) The neutron wave functions are shown in figure 12.

Replacing the classical quantities in eq. (I-5) by their quantum analogues,

$$\left(\text{i. e., } J \to \frac{J_{op}}{\sqrt{J(J+1)}} \text{ and } P_\nu (\widehat{\boldsymbol{a}} \cdot \widehat{\boldsymbol{b}}) \to (-)^\nu \sqrt{(2a+1)(2b+1)}\right.$$

$$\left. W(a\, \nu\, cb;\, ab) \text{ with } \boldsymbol{a} + \boldsymbol{b} = \boldsymbol{c}\right),$$

and noting also that the polarization vector $\boldsymbol{P}$ is defined as the expectation value of $\boldsymbol{s}_p$ divided by the maximum spin projection, one finds the final result:

$$P = \frac{1}{s_p} \cdot \left(\frac{s_p (s_p+1)}{l_n (l_n+1)} \right)^{1/2} \cdot [(-) ((2 s_p + 1) (2 s_n + 1))^{1/2} W(s_n \, 1 \, s_d \, s_p ; s_n \, s_p)] \cdot$$

$$\cdot [(-) ((2 s_n + 1) (2 l_n + 1))^{1/2} W(s_n \, 1 \, j_n \, l_n ; \; s_n \, l_n)] \, . \, \langle (\boldsymbol{l}_n)_{op} \rangle =$$

$$= \frac{1}{3} \begin{Bmatrix} l_n/(l_n+1) \text{ for } j_n = l_n + 1/2 \\ -1 \text{ for } j_n = l_n - 1/2 \end{Bmatrix} \cdot \frac{1}{l_n} \langle (\boldsymbol{l}_n)_{op} \rangle . \qquad \text{(I–6)}$$

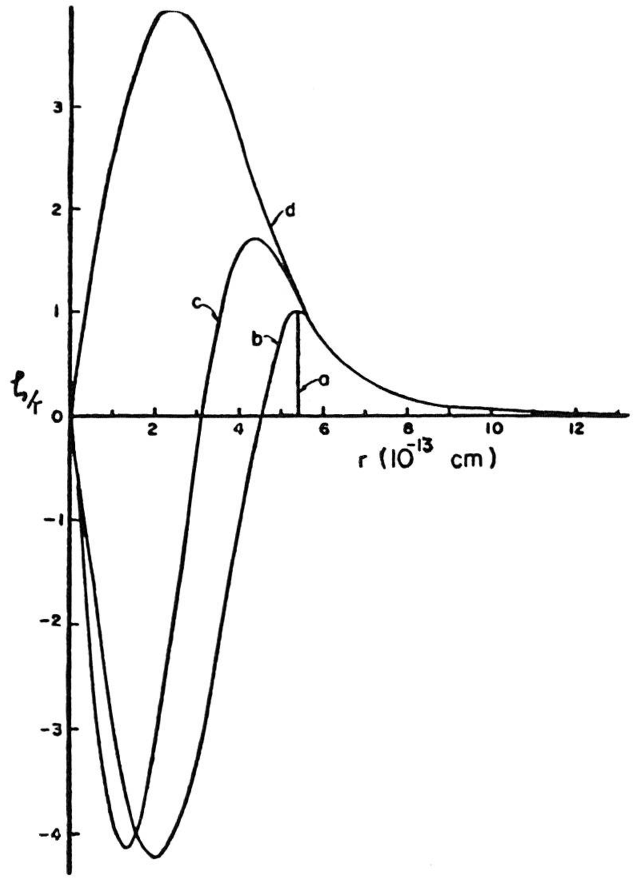

Figure 12
The radial wave functions for the captured neutron used in calculating the results shown in Figure 11.

Appendix II

Using the distorted wave Born approximation ('direct reaction') the angular distribution for the (d, p) reaction is most easily treated in the angular momentum representation, and this leads to a Legendre series for the proton distribution. This is not a particularly satisfying way to develop the answer—although it seems necessary—for the angular distribution is characteristically peaked at or near the forward direction, and requires (especially for the Coulomb field) a great many terms in the Legendre series. (One might expect that the use of an exact summation of the high l terms could be used to advantage, but this has not yet been carried out except for $l_n = 0$.)

For convenience we make two further approximations: a) a zero range $n-p$ potential and b) we take the target nucleus to be heavy, so that recoil can be neglected.

With these assumptions, the usual techniques [18] lead to the result:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\lambda\!\!\!^{-2}{}_d}{2\,s_d+1}\right)\left(\frac{2\,j_f+1}{2\,j_i+1}\right)\sum_{\nu} B_\nu\, P_\nu\,(\widehat{\boldsymbol{k}}_p \cdot \widehat{\boldsymbol{k}}_d)\,, \qquad \text{(II–1)}$$

where:

$$B_\nu \frac{1}{4} \sum \bar{Z}\,(l_p\, j_p\, l_p'\, j_p';\, s_p\, \nu)\,(-)^{\,j_d - s_d}$$

$$\bar{Z}\,(l_d\, j_d\, l_d'\, j_d';\, s_d\, \nu)\,(-)^{\,j_d - s_d}\,.$$

$$\cdot\,(-)^{\nu}\,[(2\,j_p+1)\,(2\,j_p'+1)\,(2\,j_d+1)\,(2\,j_d'+1)]^{1/2}\,W(j_p'\,\nu\,j_n\,j_d;\; j_p\,j_d')\,\cdot$$

$$\cdot\,A^{\,l_n\, j_n}_{\,l_p\, j_p;\; l_d\, j_d}\quad \cdot\,A^{*\,l_n'\, j_n}_{\,l_p'\, j_p';\; l_d\, j_d}\,, \qquad \text{(II–2)}$$

and,

$$A^{\,l_n\, j_n}_{\,l_p\, j_p;\, l_d\, j_d} = exp\; i[\delta\,(l_p\, j_p) + \delta\,(l_d\, j_d)]\; C^{\,l_d\, l_n\, l_p}_{\,0\;\,0\;\,0}\,.$$

$$\cdot\sqrt{(2\,l_n+1)\,(2\,s_n+1)}\;X\begin{pmatrix} l_d & s_d & j_d \\ l_n & s_n & j_n \\ l_p & s_p & j_p \end{pmatrix}\cdot\, G^{\,l_n\, j_n}_{\,l_p\, j_p;\; l_d\, j_d}\,. \qquad \text{(II–3)}$$

In these formulas, the symbols have their usual significance: l = orbital —, s = spin —, j = total — angular momenta; the $\bar{Z}$ coefficient is defined as in ref. [18] (except that the bar denotes the phase factor is omitted); the δ's are the phase shifts suffered by the respective proton and deuteron waves; and $C^{\cdots}_{\cdots}$ and $X(\ldots)$ denote the Wigner coefficient and (9—j) symbol respectively. The quantity $G^{\cdots}_{\cdots}$ denotes the (cut-off) radial integral over the neutron, proton, and deuteron (considered as a point particle) radial (real) wave functions. This quantity is defined exactly as is the many treatments of the stripping problem in this approximation.

The *structure* of this result, as far as angular correlation effects are concerned, is that of a general (d, p) nuclear reaction in which j_p and j_d play the role of intermediate 'nuclear' states connected by an unobserved 'radiation' of angular momentum j_n. The coefficient $A^{\cdots}_{\cdots}$ plays the role of the S matrix, and involves a 're-coupling' from $(l-s)$ to $(j-j)$ coupling for the nuclear process.

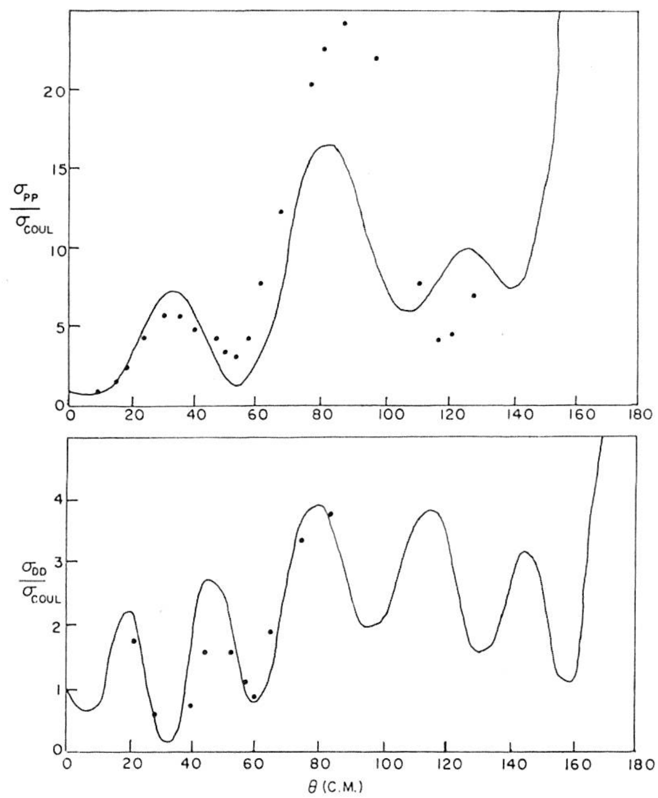

Figure 13

The elastic scattering of 20.9 MeV protons on O^{16} (top curve) and 19.0 MeV deuterons on O^{16} (bottom curve) calculated from the optical potentials listed under case d in table I. (The experimental data are from [19, 20].

The assumption that $l_n = 0$, and that the spin orbit effects on the deuteron may be neglected greatly simplifies the above result. After a little angular momentum algebra, one obtains:

$$\left(\frac{d\sigma}{d\Omega}\right)_{l_n=0} = \left(\frac{\lambda^2{}_d}{2\,s_d+1}\right)\cdot\left(\frac{2\,j_f+1}{2\,j_i+1}\right)\sum_\nu B_\nu\,(l_n=0)\;P_\nu\,(\widehat{\boldsymbol{k}_p}\cdot\widehat{\boldsymbol{k}_d})\,,$$

$$B_\nu\,(l_n=0) = \left(\bar{Z}\left(l\,j\,l\,j;\,\frac{1}{2}\,\nu\right)\right)^2 A_{lj}\,A^*_{lj}\,,$$

$$A_{lj} = e^{i\left(\delta^{(p)}_{lj}+\delta^{(d)}_{l}\right)}\;G^{0,\,1/2}_{lj;\,l}\,. \qquad \text{(II—4)}$$

Here l and j refer to the proton's orbital and total angular momentum. Since $l_n = 0$, both l_p and l_d are the same.

Appendix III

To put eq. (12) into the form given by SATCHLER, one needs the identity:

$$C^{l\,l'\,\nu}_{0\,0}\, X \begin{pmatrix} l & j & 1/2 \\ l' & j' & 1/2 \\ \nu & \nu & 1 \end{pmatrix} = \frac{(-)^{\,l'-j'+1/2}}{2\sqrt{3\,\nu(\nu+1)}} \cdot \qquad \text{(III–1)}$$

$$\cdot\,[(2\,j'+1) + (-)^{j+j'+\nu}\,(2\,j+1)]\quad C^{l\,l'\,\nu}_{0\,0} \times \begin{pmatrix} l & j & 1/2 \\ l' & j' & 1/2 \\ \nu & \nu & 0 \end{pmatrix} =$$

$$= \frac{-\,C^{l\,l'\,\nu}_{0\,0}\; W(l\,j\,l'\,j';\,1/2\,\nu)\cdot[(2\,j'+1)+(-)^{j+j'+\nu}(2\,j+1)]}{2\sqrt{6\,\nu\,(\nu+1)\,(2\,\nu+1)}}\,.$$

Appendix IV

In order to show that the polarization for small spin-orbit coupling is approximately given by the derivative of the angular distribution we assume that eq. (II-4) applies to the process at hand and then use eq. (12) to obtain the polarization.

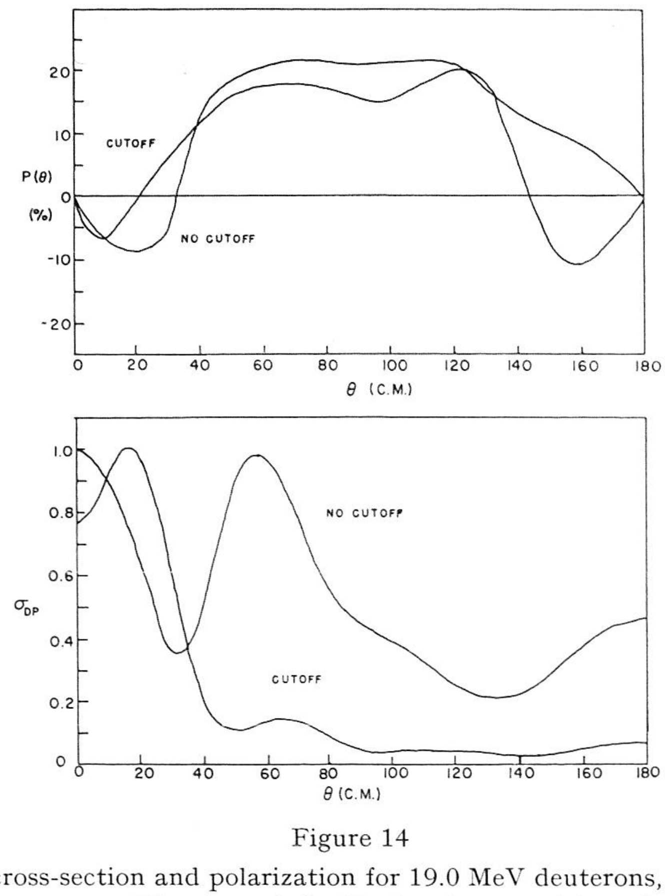

Figure 14

$O^{16}(d, p)O^{17}$ cross-section and polarization for 19.0 MeV deuterons, calculated from the distorted wave modification of the Butler theory, with and without cut-off. The various parameters are given under case *d* in table I.)

We further assume that the spin-orbit phase shift splitting is small, so that:

$$A_{lj} \cong e^{i\left(\delta_l^{(p)} + \delta_l^{(d)}\right)} G^{0,\,1/2}_{l;\,l} \left[1 + (i \varDelta \delta_{lj}^{(p)})\right] , \qquad \text{(IV–1)}$$

with $\varDelta \delta_{lj}^{(p)}$ = the spin-orbit proton phase shift splitting

$$= \beta_l \cdot \sqrt{6\, l\, (l+1)\, (2\, l+1)} \cdot W\, (l\, 1\, j\, 1/2;\, l\, 1/2) , \qquad \text{(IV–2)}$$

and

$$\beta_l \equiv - \frac{2}{k}\, e^{-2i\,\delta_l} \int_0^\infty dr\, u_l^{(-)*}(r)\, V_{\text{spin}}(r)_{\text{orbit}}\, u_l^{(+)}(r) . \qquad \text{(IV–3)}$$

(This definition of β_l is chosen to agree exactly with that used by RODBERG. His choice of radial wave function (in IV-3) differs from ours in that the phase shift $e^{i\,\delta_l}$ is included.)

(It should be noted explicitly that eq. (IV-1) *assumes* that the radial matrix elements $G^{\cdot\,\cdot}_{\cdot\,\cdot}$ are *not* effectively different with or without the spin orbit coupling. This assumption seems reasonable in itself, but is actually better than may appear since the variation in G is *real*, and thus leads to only off-diagonal contributions which are small under the same assumption that β_l is slowly varying in l.)

Inserting (III-1, 2) into eq. (12), (using (II-4) for the definition of B_ν) one finds for the polarization the expression:

$$\boldsymbol{P}(\theta) = \hat{\boldsymbol{n}}\, \frac{\lambda\!\!\!^{-2}{}_d\, (2\, j_f+1)}{(2\, s_d+1)\, (2\, j_i+1)} \left(\frac{d\sigma}{d\Omega}\right)^{-1} \sum_{\nu l l'} 2\, e^{i\left[\delta_l^{(p)} + \delta_l^{(d)} - \delta_{l'}^{(p)} - \delta_{l'}^{(d)}\right]} \,\cdot$$

$$\cdot \frac{1}{4} \left(G^{0,\,1/2}_{l,\,l}\right)^2 \beta_l \sqrt{6\, l\, (l+1)\, (2\, l+1)} \left(C^{\,l\, l'\, 0}_{\,0\, 0}\right)^2 (2\, l+1)\, (2\, l'+1)$$

$$\frac{d}{d\theta} \cdot (P_\nu (\cos\theta)) \sqrt{\frac{6(2\,\nu+1)}{\nu(\nu+1)}} \left\{ \sum_{j j'} (-)^{\nu + l' + j' + 1/2}\, (2\, j+1)\, (2\, j'+1) \,\cdot \right.$$

$$\left. \cdot\, X \begin{pmatrix} l & j & 1/2 \\ l' & j' & 1/2 \\ \nu & \nu_1 & \end{pmatrix} W \left(l\, j\, l'\, j';\, \frac{1}{2}\, \nu\right) \; W \left(l\, 1\, j\, \frac{1}{2}\, ;\, l\, \frac{1}{2}\right) \right\} .$$

(This employs the result that $- d\, P_\nu / d\,\theta = P_\nu^{(1)}$.) (IV–4)

The bracketed sum in eq. (IV-4) may be carried out using the various sum rules given in ref. [18]. The result is:

$$\left\{ \cdots \right\} = \frac{1}{3}\, W\, (l\, 1\, l'\, \nu;\, l\, \nu) . \qquad \text{(IV–5)}$$

Inserting this result (in its explicit algebraic form) in eq. (IV-4) one finds the desired result:

$$\boldsymbol{P}(\theta) = \hat{\boldsymbol{n}} \left(\frac{d\sigma}{d\Omega}\right)^{-1} \frac{\lambda\!\!\!^{-2}{}_d(2\, j_f+1)}{(2\, s_d+1)\,(2\, j_i+1)} \cdot \frac{d}{d\theta} \sum_{\nu l l'} \cdot$$

$$\cdot\, \beta_l \left[\frac{1}{4}\,(2\, l+1)\,(2\, l'+1)\left(C^{\,l\; l'\nu}_{\,0\; 0}\right)^2 A_l\, A_l^*\, P_\nu\,(\cos\theta)\right]. \qquad \text{(IV-6)}$$

If we assume, following RODBERG, that it is justified to regard β_l as slowly varying with l so that $\beta_l \simeq \langle\beta_l\rangle$ then one finds that (IV-6) takes the form:

$$\boldsymbol{P}(\theta) \simeq \frac{1}{2}\, \hat{\boldsymbol{n}}\, (\langle\beta_l\rangle)\, \frac{d}{d\theta}\left(\frac{d\sigma}{d\Omega}\right) \Big/ \frac{d\sigma}{d\Omega}. \qquad \text{(IV-7)}$$

(The further approximation that $\langle\beta_l\rangle \simeq -\frac{mR}{\hbar^2 k} V_{so}\,(R)$,

as given in ref. [11], may be of value in obtaining a rough estimate of the spin orbit potential.)

Appendix V

If, in addition to the usual approximation of the direct reaction process with a zero range $n-p$ potential, one neglects all except Coulomb distortions and takes the nucleus to have $R_n = 0$, the stripping integral may be written in the form (for $l_n = 1$):

$$\boldsymbol{M} = \int d^3\, \boldsymbol{r}\, \hat{\boldsymbol{r}}\, h_1(i\, k_n\, r)\, \psi_p^*\,(\boldsymbol{r})\, \psi_d(\boldsymbol{r})$$

where:

$$\psi_p^*(\boldsymbol{r}) = \left(\frac{2\,\pi\,\eta_p}{e^{2\,\pi\,\eta_p}-1}\right)^{1/2} e^{-i\,\boldsymbol{k}_p\cdot\boldsymbol{r}}\; {}_1F_1\,(-\,i\,\eta_p,\, 1;\, i\,(k_p\, r + \boldsymbol{k}_p\cdot\boldsymbol{r}))$$

$$\psi_d(\boldsymbol{r}) \equiv \left(\frac{2\,\pi\,\eta_d}{e^{2\,\pi\,\eta_d}-1}\right)^{1/2} e^{i\,\boldsymbol{k}_d\cdot\boldsymbol{r}}\; {}_1F_1\,(-\,i\,\eta_d,\, 1;\, i\,(k_d\, r - \boldsymbol{k}_d\cdot\boldsymbol{r})\,. \qquad \text{(V-1)}$$

(All factors independent of k_p, k_d, η_p, η_d, k_n have been eliminated.)

The approximation that $R_n = 0$ is made for convenience, so that SOMMERFELD's techniques may be applied to evaluating this integral. This approximation is of little importance when the reaction is Coulomb-limited; for cases where this is not applicable a rapidly convergent series for the contribution from $0 \leqq r \leqq R_n$ may be subtracted explicitly from the result in eq. (V-1). The usefulness of the results given below (or in ref. [15] for $l_n = 0$) lies in the fact that the contributions for large values of r and l are treated concisely.

With the application of SOMMERFELD's techniques this integral may be written as:

$$\boldsymbol{M} = (16\,\pi^2\,i) \left[\frac{\eta_p\,\eta_d}{(e^{2\pi\eta_p}-1)\,(e^{2\pi\eta_d}-1)}\right]^{1/2} \cdot$$

$$\cdot \int_{k_n}^{\infty} ds\,(s^2 + k_p^2 + k_d^2 - 2\,k_p\,k_d\cos\theta)^{-1-i\eta_p-i\eta_d} \cdot$$

$$\cdot\,(k_d + k_p + is)^{-1+i\eta_p+i\eta_d}\,(k_d - k_p - is)^{-1+i\eta_p} \cdot$$

$$\cdot\,(k_p - k_d - is)^{-1+i\eta_d}\quad\{\cdots\}\,, \tag{V–2}$$

$$\{\cdots\} \equiv \left\{\left[\frac{s^2+(k_d-k_p)^2}{s^2+(\boldsymbol{k}_d-\boldsymbol{k}_p)^2}\,(1+i\,\eta_p+i\eta_d)\,(k_p+k_d+is)\,(\boldsymbol{k}_d-\boldsymbol{k}_p)\,+\right.\right.$$

$$+\,s\,(\eta_p\,\boldsymbol{k}_d - \eta_d\,\boldsymbol{k}_p) + i\,(k_p-k_d)\,(\eta_d\,\boldsymbol{k}_p+\eta_p\,\boldsymbol{k}_d)\Big]\,F\,+$$

$$+\,2\left[(k_p\,\boldsymbol{k}_d - k_d\,\boldsymbol{k}_p) + \frac{2\,k_p\,k_d\sin^2\frac{\theta}{2}}{s^2+(k_d-k_p)^2}\,\times\right.$$

$$\left.\left.\times\,i\,s\,(\boldsymbol{k}_p-\boldsymbol{k}_d) + (k_p-k_d)\,(\boldsymbol{k}_p+\boldsymbol{k}_d)\right]F'\right\}, \tag{V–3}$$

where:

$$F = {}_2F_1\left(-i\,\eta_p,\,-i\,\eta_d,\,1;\left(\frac{-4\,k_p\,k_d\sin^2\frac{\theta}{2}}{(k_p-k_d)^2+s^2}\right)\right),$$

and

$$F' \equiv \frac{d}{dx}\left({}_2\mathrm{F}_1\,(a,b,c;x)\right). \tag{V–4}$$

Although this integral is hardly simple, nonetheless it does present certain information. For example, when $|\boldsymbol{k}_p| = |\boldsymbol{k}_d|$ there are but two combinations of the vectors $\boldsymbol{k}_p$ and $\boldsymbol{k}_d$ that enter: $(\boldsymbol{k}_d - \boldsymbol{k}_p)$ and $(\eta_p\,\boldsymbol{k}_d - \eta_d\,\boldsymbol{k}_p)$. From this one sees that the polarization varies as $(\eta_d - \eta_p)\,(\boldsymbol{k}_d \times \boldsymbol{k}_p)$ and is thus dominated by the deuteron distortion. (The special case where $k_p = k_d$ is amenable to further reduction, but these results will be given elsewhere.)

Since the integral $\boldsymbol{M}$ decreases essentially exponentially as k_n increases, it is clear also that the integration over s is in effect only the definition of an average value of k_n. Qualitatively then, one may simply dispense with the integral and take $s = \langle k_n \rangle$. Next one notes that $F' \approx \eta^2$ while $F \approx 1$ so that $\{\ldots\}$ may, for small η, be approximated by the first $[\ldots]$ bracket. With these simplifications one obtains for the polarization:

$$P \cong \frac{2}{3}\, \hat{\boldsymbol{n}} \begin{Bmatrix} 1/2 \\ -1 \end{Bmatrix} \sin\vartheta \left(\frac{k_n^2 + (k_d - k_p)^2}{k_n^2 + (k_d - k_p)^2 + 4\, k_p\, k_d \sin^2 \frac{\vartheta}{2}} \right) \frac{1}{|\{\ldots\}|^2} \cdot$$

$$\cdot\, [k_n^2\,(\eta_d - \eta_p) + (k_p^2 - k_d^2)\,(\eta_d + \eta_p)]. \qquad \text{(V-5)}$$

(small η approximation)

(the bracket $\{\ldots\}$ in (V-5) is the same as in (V-3); for $k_p = k_d$ and $\theta \to 0$, the terms $\theta(\eta^2)$ must be kept in the denominator).

One sees from this approximate result that the polarization is positive (parallel to $\boldsymbol{k}_d \times \boldsymbol{k}_p$) for $k_d \approx k_p$, but can become negative as k_p gets sufficiently greater than k_d. In general, one would expect this approximate result to be qualitatively valid in the forward direction, and the examples discussed seem to bear this out.

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