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## Selection Rules for Polarization in Direct Interactions and Stripping Processes

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There exists, by now, quite a variety of direct nuclear reactions. Of these, the most extensively studied is deuteron stripping and its associated pick-up process. Recent studies have been concerned with the stripping of two nucleons,  $\text{He}^3$  or  $\text{H}^3$ , and pick-up phenomena such as  $(p, \alpha)$ ,  $(d, \alpha)$ ,  $(\text{He}^3, \alpha)$ . Then, there is the phenomenon of heavy-particle stripping where the target nucleus is stripped instead of the projectile. Finally, we have the inelastic scattering of nucleons or heavier particles. All these processes are termed direct so as to create a distinction, with the competitive processes that go by compound-nucleus formation. For these latter reactions, in principle, we need many parameters to give a proper description of the many degrees of freedom of the nuclear system. In contrast to this, direct interactions are typified by very few parameters. These essentially are the quantum numbers associated with a transfer of angular momentum and energy between the system of incoming and outgoing projectiles and that of the target and residual nuclei. If the process is to be easily handled, we expect only a few quantum numbers to play any significant role.

It is convenient for our purposes to illustrate the processes by means of angular-momentum graphs,

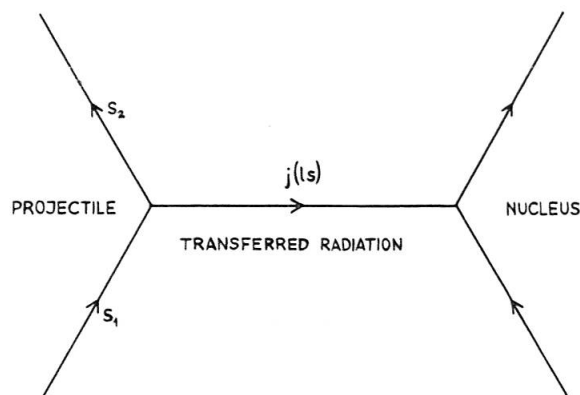


Figure 1

e.g. for  $d$ - $p$  processes  $s_1 = 1$  and  $s_2 = s = 1/2$ . The transferred radiation consists of neutrons. On the other hand, for inelastic scattering, there is no exchange of real particles. Rather, the interaction operator can be made to factorise into a part acting on the nuclear configuration and another part acting on the projectile system. These factors are tensors, contragredient to each other, which behave under rotations just like angular-momentum operators. The quantities  $j$ ,  $l$  and  $s$ , in fact, incorporate the properties of these tensors under rotation.

To deal with the de-excitation of the residual nucleus, we alter our diagram

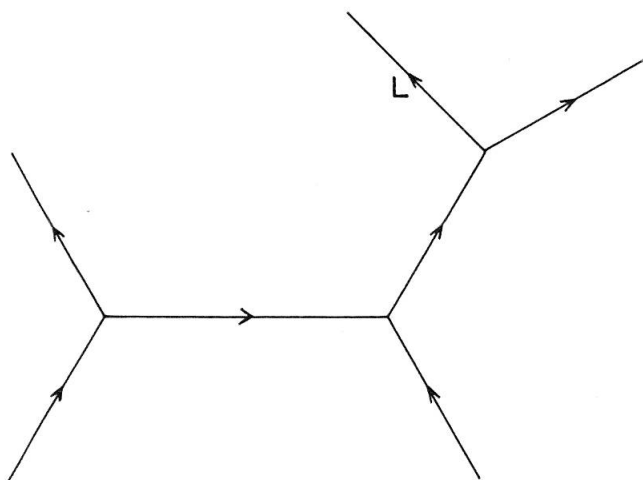


Figure 2

so that  $L$  denotes the multipolarity of the  $\gamma$ -radiation.

In principle, measurement of the angular correlations here serves as a measure of the polarization state of the residual nucleus. To typify polarization measurements of the projectiles, we alter the diagram symbolically by breaking up the angular-momentum states of the projectiles into those of orbital angular momentum and intrinsic spin.

It is possible to handle all these possibilities in a systematic fashion by means of the modern analytical techniques of dealing with angular-momentum decoupling, and since these diagrams can be made to apply to all the direct processes, we can treat these together.

The earliest calculations were done in Born approximation. Although, in many cases, particularly in stripping, the angular distributions were well-approximated, the same does not hold for polarization. As is well-known, the Born approximation predicts zero polarization in contrast to large values obtained by more accurate procedures.

These procedures involve use of the distorted-wave Born approximation, where the interactions between the projectiles and nuclei in the incident and outgoing channels are approximated by the use of

optical-model potentials, derived from a study of elastic scattering. However, if one ignores the spin-orbit term in the potential, one finds several restrictive rules for polarization. Thus, as originally found by SATCHLER for deuteron-stripping, second-rank (i.e. tensor) polarization of the deuteron gives rise to no asymmetry in the angular distribution. Also the effects of 1st rank (i.e. vector) polarization are directly correlated to the polarization that would be measured in an identical experiment but where the incoming deuteron-beam is unpolarized. Finally, if  $l=0$ , we get no polarization nor effects of polarization. This is independent of the complication of the distortion, so long as this distortion is spin-independent.

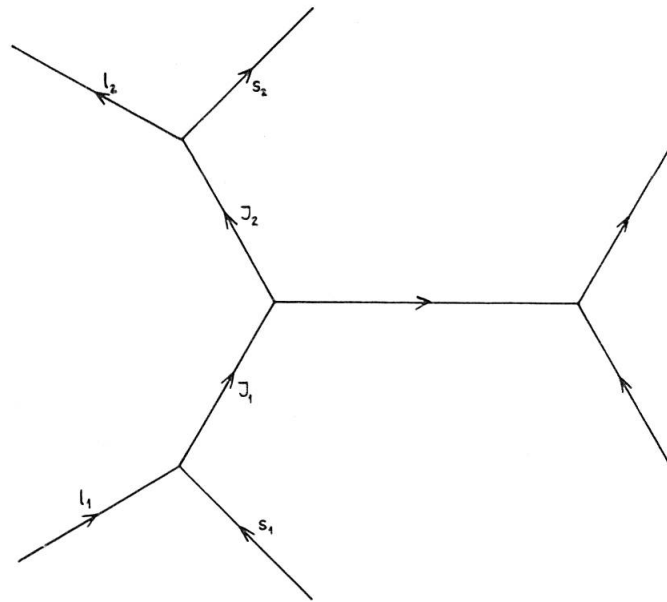


Figure 3

Calculations in this approximation require very elaborate computing programs and for this reason the progress to-date is restricted. TOBOCMAN recently reported the results of some of his analyses of several deuteron stripping processes where he finds in certain cases significant contributions to the cross-sections and, in particular, polarization through the introduction of distorted waves. The same sensitivity has been noticed in numerical calculations of inelastic nucleon scattering. TOBOCMAN ignores the spin-orbit part of the distorting potential in the belief that its magnitude is small compared to the central part, so that its effect should be minor. Still, one should be careful with such a procedure since it is not clear that the effect of the spin-orbit contribution should be the same in elastic scattering and in the quite-different stripping problem. Because of the sensitivity of the polarization to the distortion, it would seem to be safe practice to try and relate the parameters as close as it is possible to

those established by the elastic scattering analyses. The complications, of course, that are added to the computation are great, and would require elaborate use of electronic computers.

I'd like to mention in this context, some calculations we did with BROWN and CASTILLEJO from Birmingham, concerning the inelastic scattering of protons by Ni<sup>58</sup> for various energies about 9 MeV and higher. Polarization would vanish in the absence of a spin-orbit potential term in the distortion; yet by including this relatively insignificant term, we found polarizations which for certain angles exceeded 50%. At higher energies  $\sim 15$  MeV, polarizations greater than 75% were found.

With this sensitivity of polarization to the spin-dependent distortion in mind, we have arrived at general expressions for angular distributions and polarization for arbitrary types of beams. Certain general rules are obtained which depend on the presence of this distortion.

We deal with a general process with spins  $s_1$  and  $s_2$  for the projectiles and  $s$  for the transferred radiation. We make the idealisation that in dealing with composite particles, there is no coupling of the spin to the internal configuration of the particle. This refers to the projectiles or the transferred radiation. Thus, for example, we ignore contributions from the  $d$ -state of the deuteron. Also, we ignore any spin-dependence of the direct-reaction potential. This restriction which is not meant to apply to inelastic scattering, is not too serious since to-date only the crudest types of direct interactions have been used, viz., delta-function interactions in coordinate space.

In general, no selection rules are apparent. Of interest, though from the point of view of numerical computation, is the fact that the angular dependence is best expressed in terms of the elements of the rotation matrix,  $D_{\lambda\lambda'}^l(0, \theta, 0)$ , and not the usual spherical harmonics. This is associated with the introduction of spin-orbit contributions and applies both to cross sections and to polarization. We consider either measurement of the outgoing polarization using unpolarized incident beams, or the asymmetries arising from incident polarized beams. We then limit the spin-dependent distortion to either incoming or outgoing stage or to neither stage.

A particle of spin  $s$  has polarization tensors of rank  $k$  ranging from 0 up to  $2s$ . If we let  $k_1$  denote the rank of the incident polarization that is being measured and  $k_2$  that of the outgoing polarization that will be measured, then our first general result is that for no spin-dependent distortion  $k_1, k_2 \leq 2l_{max}$  or  $2s_{max}$  e.g.  $l = 0$ , gives us neither polarization nor effects of polarization.

Treating  $l = 0$ , and letting spin-dependent distortion enter in the incoming stage, we find  $k_2 \leq 2s_1$ , while if it acts only in the outgoing stage,  $k_1 \leq 2s_2$ . This is all represented in the following table.

Table

No spin-dependent distortion	Spin-dependent distortion of	
	incoming beam only	outgoing beam only
$k_1, k_2 \leq 2 s_{max}$ $\leq 2 l_{max}$		
$(l = 0): k_1 = k_2 = 0$	$k_2 \leq 2 s_1$	$k_1 \leq 2 s_2$
<i>Deuteron stripping</i>		
$(l \neq 0): A = 1 + 3 P_{in} P_{out}$ $(l = 0): = 1$	$1 + P_{in} P_{out}$	$1 + 3/2 P_{in} P_{out}$
$ P_{out}  \leq 1/3$ $j = l - 1/2$ $\leq 1/3(l+1)$ $j = l + 1/2$	$ P_{out}  \leq 1$	
<i>(He<sup>3</sup>, p)-(H<sup>3</sup>, p)-processes etc.</i>		
$(s = 0): A = 1$ $(s = 1) (l \neq 0): = 1 - P_{in} P_{out}$ $(l = 0): = 1$	$1 + P_{in} P_{out}$ $1 - 3 P_{in} P_{out}$	$1 + P_{in} P_{out}$ $1 - 1/3 P_{in} P_{out}$
$(s = 0):  P_{out}  = 0$ $(s = 1):  P_{out}  \leq l/(l+1)$ $j = l+1$ $\leq 1/(l+1)$ $j = l$ $\leq 1$ $j = l-1$		
<i>(<math>\alpha</math>, p)-processes</i>		
$(l \neq 0):  P_{out}  \leq 1$ $j = l - 1/2$ $\leq l/(l+1)$ $j = l + 1/2$ $(l = 0): P_{out} = 0$		
<i>(<math>\alpha</math>, d)-processes</i>		
$ P_{out}  \leq l/(l+1)$ $j = l+1$ $\leq 1/(l+1)$ $j = l$ $\leq 1$ $j = l-1$		
<i>Inelastic nucleon-scattering</i>		
$A \approx 1 + P_{in} P_{out}$	assuming spin-dependent distortion of incoming and outgoing beam.	

Thus, for deuteron stripping, there are no second rank effects with no spin-orbit distortion for arbitrary  $l$  since  $s = 1/2$ , while if  $l = 0$ , polarization effects do not show up for no spin-distortion. If, however, there is a spin-dependent distortion acting only on the nucleon, polarization effects occur but these are again limited to first rank only.

Correlations between polarizations can also be established. Suppose in deuteron-stripping,  $P_{in}$  is the incident deuteron polarization of 1st rank

normal to the plane of the process. Then the angular distribution is

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right) A = \left(\frac{d\sigma}{d\Omega}\right)_0 [1 + 3 P_{in} P_{out}], \quad \text{where } \left(\frac{d\sigma}{d\Omega}\right)_0$$

is the unpolarized cross-section, and  $A$ , the correlation, takes on the value  $1 + 3 P_{in} P_{out}$  for no spin-dependent distortion. The coefficient,  $P_{out}$ , is the outgoing polarization as measured in another experiment under identical conditions, except for an unpolarized incident beam. Thus, if in one experiment the incoming polarization is  $P_{in}$ , in the other experiment, the left-right asymmetry is equal to  $3 P_{in}$ . The correlation  $A$  for  $l = 0$  and spin-dependent distortion in the incoming or outgoing stage or in neither takes on the values  $1 + P_{in} P_{out}$ ,  $1 + 3/2 P_{in} P_{out}$ , or 1 respectively.

It is clearly of interest, then to examine polarization of  $l = 0$  stripping processes.

As already pointed out, the outgoing polarization is never greater than  $1/3$  for no spin-dependent distortion (actually only for  $j = l - 1/2$ , but for  $j = l + 1/2$ ,  $|P_{out}| \leq 1/3 l/(l + 1)$ ). This limitation is removed if the distortion is generalised.

Several selection rules obtain when we treat the stripping of two nucleons e.g.  $(\text{He}^3, p)$ ,  $(\text{H}^3, p)$  etc. Here,  $s = 0$  or 1, although for  $\text{H}^3$ , processes we expect only  $s = 0$ . First consider  $s = 0$ . Then in the absence of spin-dependent distortion, no polarization effects occur at all, while if this type of distortion acts on only one stage, the correlation function is  $A = 1 + P_{in} P_{out}$ . This is for arbitrary  $l$ .

For  $s = 1$ , polarization is present even in the absence of spin-dependent distortion where in this case  $A = 1 - P_{in} P_{out}$  except that  $l = 0$  again gives us no polarization. With  $l = 0$  and  $s = 1$  and spin-dependent distortion in the incoming or outgoing stage only  $A = 1 - 3 P_{in} P_{out}$  or  $1 - 1/3 P_{in} P_{out}$  respectively. Again for  $s = 1$  and single  $j$  and  $l$ , then  $|P_{out}| \leq l/(l + 1)$ ,  $1/(l + 1)$ , 1 according as  $j = l + 1$ ,  $l$ ,  $l - 1$  in the absence of spin-dependent distortion.

Furthermore, we no longer necessarily have incoherence over  $l$  and  $s$ . In all these processes, the summation over  $j$  is incoherent, so long as we do not measure the polarization state of the target or residual nuclei. The summation over  $l$  is also incoherent for deuteron stripping since  $l = j \pm 1/2$  and if two  $l$ 's interfere and differ by one, they involve a different parity change for the nuclear system which is not possible. However if  $s = 1$ ,  $l = j \pm 1$ ,  $j$  and the  $l$ 's can differ by two and contribute coherently. This coherence occurs only if the distortion is spin-dependent.

We can continue our observations and refer to  $(\alpha, d)$ ,  $(\alpha, \text{He}^3)$   $(\alpha, p)$  processes. Polarization occurs for  $l = 0$ , only as a consequence of spin-dependent distortion; but for  $l \neq 0$ , it occurs without this type of distur-

tion. Also for  $(\alpha, d)$  processes the summation over  $l$  is generally coherent. In the absence of spin-dependent distortion  $|P_{out}| \leq \frac{l(l+1)}{1}$  for  $(\alpha, p)$ ,  $(\alpha, \text{He}^3)$  processes according as  $j = l + 1/2$  or  $l - 1/2$  as found by SATCHLER. For  $(\alpha, d)$  processes  $|P_{out}| \leq l/(l+1), 1/(l+1), 1$  according as  $j = l + 1, l, l - 1$ .

Now all these rules, except for the question of coherence in  $l$  apply whether or not parity is conserved or time-reversal invariance holds. If we invoke time-reversal invariance, then we get an interesting result for inelastic scattering of nucleons. We find that  $\mathcal{A}$  is approximately  $1 + P_{in} P_{out}$ . The difference arises in the radial integration that is involved. The outgoing polarization differs from the asymmetry in that the optical-model eigenfunctions appear in an identical fashion, except for a switching of energies. If the incoming and outgoing optical-model eigenfunctions describing the distortion are not too sensitive to the difference of incoming and outgoing energies, the above result holds. In the limit of no energy loss, we have true equality and the result (called for nucleon-nucleon scattering the polarization-asymmetry theorem) is independent of parity conservation. The same holds true for inelastic scattering. Of practical importance, is the fact that we get little new information by bringing in polarized nucleons as compared to that obtained by measuring the outgoing polarization, since the effects are almost completely correlated.

Finally some remarks about  $\gamma$ -correlations. We find that although correlation studies can give a great deal of independent information, such measurements with  $l = 0$  and  $s = 0$  or  $1/2$  give no information at all if the  $\gamma$ -radiation is of sharp parity. On the other hand if  $l \neq 0$ , we get contributions with a coherent summation over  $j$  and  $l$  even if  $s = 0$  or  $1/2$ . In certain cases observation of the complexity of the angular correlation, i.e. the largest value of  $k$  in the expansion in terms of  $P_k(\cos\theta)$ , can single out effects due to  $s = 1$ , for example in the  $(\text{He}^3, p)$  processes.