Zeitschrift: Helvetica Physica Acta

Band: 38 (1965)

Heft: I

Artikel: Symmetry breaking in the static strong-coupling theory

Autor: Wentzel, G.

DOI: https://doi.org/10.5169/seals-113575

Symmetry breaking in the static strong-coupling theory*

by **G. Wentzel**

Enrico Fermi Institute for Nuclear Studies and
Department of Physics, The University of Chicago, Chicago, Illinois**)

(21. IX. 64)

Abstract. The question is studied how the energy spectra of the nucleon (or baryon) 'isobars' are affected if differences in meson masses are assumed which violate either charge symmetry or $SU(3)$ symmetry. Scalar theories show a high sensitivity to such symmetry breaking causes. The situation is markedly different in a pseudoscalar theory (nucleon and pions) where the charge symmetry appears stabilized through the interdependence of spin and isospin ($J = I$).

1. Introduction

The mass formula of GELL-MANN[1]) and OKUBO[2]) has recently stimulated much interest in "broken symmetries". While the raison d'être both of higher symmetries and their violations, in particle physics, remains mysterious, all theoretical implications deserve to be explored.

The following special question will be studied here, in the framework of the static strong-coupling approximation[3]): Given certain primary symmetry violations in the mass spectrum of the (bare) mesons, in what way and how strongly will they affect the mass spectrum of the baryon-meson bound states or resonances ("isobars")?

Compared with the more popular dispersion-theoretic methods, the static strong-coupling theory has the virtue that it reduces such problems to clearly defined wave-mechanical problems. In a realistic interpretation, of course, allowance must be made for the over-idealized nature of the approximation.

Once the wave-mechanical problem is formulated, a qualitative discussion of the solutions will often be sufficient. Also, some simplest models will already indicate what may happen in mathematically more complicated cases.

2. Nucleon and Scalar Mesons

To introduce our first model, we write out the Hamiltonian of an "almost charge-symmetric" scalar field theory:

$$H = H^0 + H' , \tag{1}$$

$$H^0 = \frac{1}{2} \int d^3x \sum_\varrho \left[\pi_\varrho^2(x) + \psi_\varrho(x)\, (\mu_\varrho^2 - \Delta)\, \psi_\varrho(x)\right] , \tag{2}$$

*) Dedicated to ERNST C. G. STUECKELBERG on his 60th birthday.

**) This work was supported in part by the U.S. Atomic Energy Commission, Cont. AT(11-1)-264.

$$H' = g \sum_\varrho \tau_\varrho q_\varrho \, , \quad q_\varrho = \int d^3x \, v(x) \, \psi_\varrho(x) = q_\varrho^* \, . \tag{3}$$

τ_ϱ ($\varrho = 1, 2, 3$) are the Pauli matrices representing the bare-nucleon isospin ($I = {}^1/_2$, $I_3 = \pm {}^1/_2$). The form-factor $v(x)$ is normalized like a δ-function:

$$\int d^3x \, v(x) = 1 \, .$$

Note that we retain strict charge-symmetry (rotational invariance) in H' [4]) and put the violation entirely into H^0 by allowing charged and neutral mesons to have different masses:

$$\mu_1 = \mu_2 \neq \mu_3 \, .$$

The bare-nucleon energy is omitted as a constant. (Taking the bare proton and neutron masses different would have no effect since only 50-50 mixtures occur in the stationary states if $g \gg 1$ [5]).

In the strong-coupling approximation, the first step is to diagonalize the 2×2 matrix H' which contains the "large" factor g (actually, $g \gg 1$ will be required). The unitary transformation achieving this is well known:

$$U^+ H' U = g \, \tau_3 \, r = g \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} r \, , \tag{4}$$

$$r = \left(\sum_\varrho q_\varrho^2 \right)^{1/2} \; (> 0) \, . \tag{5}$$

The two eigenvalues $\pm g \, r$ are widely separated, the matrix elements of $U^+ H^0 U$ linking the two turn out to be negligible, and the low-energy eigenstates – the only ones we are interested in – are described by a statevector with only one isospinor component $F \neq 0$ (viz. the one associated with the eigenvalue $-g \, r$ of H', taking $g > 0$).

The second step is to reduce the Schrödinger equation for F to a problem of small oscillations and rotations. The Hamiltonian consists of "kinetic" and "potential" energy terms:

$$T = \left[U^+ \frac{1}{2} \int d^3x \sum_\varrho \pi_\varrho^2(x) \, U \right] \text{(diagonal part)}, \tag{6}$$

$$V = \frac{1}{2} \int d^3x \sum_\varrho \psi_\varrho(x) \, (\mu_\varrho^2 - \Delta) \, \psi_\varrho(x) - g \, r \, , \tag{7}$$

where r is given by (5) and (3). For a given direction $e_\varrho = q_\varrho / r$ of the isovector q_ϱ, V has a minimum as a functional of $\psi_\varrho(x)$. The location of this minimum is

$$\psi_\varrho^0(x) = r^0 \, e_\varrho \, Y_\varrho^{-1} \, (\mu_\varrho^2 - \Delta)^{-1} \, v(x) \, , \tag{8}$$

where

$$Y_\varrho = \int d^3x \, v(x) \, (\mu_\varrho^2 - \Delta)^{-1} \, v(x) \, , \tag{9}$$

$$r^0 = g \left(\sum_\varrho e_\varrho^2 \, Y_\varrho^{-1} \right)^{-1} , \tag{10}$$

and the value of V at this location is

$$V^0 = -\frac{1}{2} \, g \, r^0 = -\frac{1}{2} \, g^2 \left(\sum_\varrho e_\varrho^2 \, Y_\varrho^{-1} \right)^{-1} . \tag{11}$$

In the strictly charge-symmetric case (all $\mu_\varrho = \mu$, all $Y_\varrho = Y$), $r^0 = g\ Y$ becomes independent of the direction e_ϱ, in other words, the "potential valley" has spherical symmetry in the 3-dimensional q_ϱ-space, and indeed, as is well known, the lowest eigenstates correspond to rotational motions in this spherical valley, with eigenfunctions describable in terms of ordinary spherical harmonics. All this is no longer true if the charge-symmetry is broken such that

$$Y_1 = Y_2 \neq Y_3 . \tag{12}$$

The potential valley is now anisotropic, though still axially symmetric, of course. The most important point is that the *depth* of the potential valley is *strongly anisotropic*, in the sense that the deviation from the mean carries the large factor g^2.

The total field ψ_ϱ is now split into a part [essentially ψ_ϱ^0 (8)] which describes the "bound" mesons, and a part orthogonal to it, corresponding to quasi-free mesons interacting only weakly with the bound system. Then, corresponding parts must also be isolated from the "kinetic" energy (6). Techniques for doing this are well known[3]) and need not be recapitulated. Here we are concerned only with the *bound meson* part of T which is expressible as a quadratic form in $p_\varrho = -i\,\partial/\partial q_\varrho$, with coefficients acting as an "effective-mass" tensor. Although the construction of this tensor is straightforward, it will suffice for the following discussion to note the result for the *isotropic* case: $\mu_\varrho = \mu$. Introducing angular coordinates through

$$e_1 + i\,e_2 = \sin\vartheta\, e^{i\varphi},\ e_3 = \cos\vartheta,$$

one obtains for the rotational energy:

$$H_{rot} = -\frac{1}{2}\,\varepsilon\left[\frac{1}{\sin\vartheta}\,\frac{\partial}{\partial\vartheta}\,\sin\vartheta\,\frac{\partial}{\partial\vartheta} + \frac{1}{\sin^2\vartheta}\left(\frac{\partial^2}{\partial\varphi^2} + i\,\cos\vartheta\,\frac{\partial}{\partial\varphi} - \frac{1}{4}\right)\right], \tag{13}$$

$$\varepsilon = g^{-2}\left[\int d^3x\, v(x)\,(\mu^2 - \Delta)^{-2}\, v(x)\right]^{-1} . \tag{14}$$

In the point source limit $[v(x) \to \delta(x)]$:

$$\varepsilon = g^{-2}\, 8\,\pi\,\mu . \tag{15}$$

The two last terms in (13) result from the U transformation [see (6)] which also (since $U \sim e^{\pm i\varphi/2}$) enforces half-integral quantization ($-i\,\partial/\partial\varphi = I_3 = {}^1/_2 +$ integer). To (13) might be added, beside the constant V^0, the energy of the radial vibration whose frequency, however, is much larger than ε (though $\ll |\, V^0\,|$), so only the vibrational groundstate need be considered. If additive constants are absorbed into the nucleon mass, the energies of the lowest stationary states are then given by the eigenvalues of H_{rot}:

$$E = \frac{1}{2}\,\varepsilon\, I\,(I+1)\,,\quad I = \frac{1}{2},\ \frac{3}{2},\ \cdots\ ,$$

$$I_3 = \pm\, I,\ \pm\,(I-1), \ldots\quad \left[I_3 + \frac{1}{2} = \text{charge}\right]. \tag{16}$$

This summarizes the case of complete charge-symmetry.

Now let $\delta\mu = \mu_1 - \mu_3$ depart from zero. Even as a first order perturbation, the anisotropic parts of the Hamiltonian cause, for $I \geqslant 3/2$, a splitting of the energy levels according to $| I_3 |$. But the perturbation method fails as soon as this splitting becomes comparable with the "small" rotational excitation energies ($\sim \varepsilon \sim \mu\, g^{-2}$). Indeed, the spectrum is changed radically if $g^2 \mid Y_1 - Y_3 \mid$ [see (11)] becomes $\gg \varepsilon$. A simple semi-classical argument (*WKB* approximation for the ϑ motion) shows that then the wave-function shrinks into a deep potential well located either near $\vartheta = \pi/2$ or near $\vartheta = 0$ and π, depending on whether $Y_1 \gtrless Y_3$. In other words, the ϑ motion is obstructed by a potential barrier and becomes a mere *oscillation*, in the lowest states. We list the asymptotic behaviour of the energy spectra as the oscillatory frequencies

$$\nu \cong g\,(\varepsilon \mid Y_1 - Y_3 \mid)^{1/2} \tag{17}$$

become $\gg \varepsilon$:

$$\text{if } Y_1 > Y_3 \text{ (or } \mu_1 < \mu_3\text{)}: E = \frac{1}{2}\,\varepsilon_1 \left(I_3^2 + \frac{1}{4}\right) + \nu_1 \left(n + \frac{1}{2}\right), \tag{18}$$

$$\text{if } Y_1 < Y_3 \text{ (or } \mu_1 > \mu_3\text{)}: E = \nu_3 \left(\mid I_3 \mid + n + \frac{1}{2}\right), \tag{19}$$

where $n = 0, 1, 2, \ldots$, while $I_3 + {}^1/_2$ again stands for the charge. In the case (18) there is still a rotation in the plane $\vartheta = \pi/2$ and accordingly a narrow-spaced rotational spectrum, whereas in (19) the motion is reduced to two-dimensional oscillations (alternatingly near $\vartheta = 0$ and $\vartheta = \pi$ as n increases[6]), with the result that the only "low-lying" states are $I_3 = \pm\, {}^1/_2$.

In the point source limit, $v(x) \to \delta(x)$, one obtains from (9), (15), and (17):

$$\delta Y \equiv Y_1 - Y_3 = -\,\frac{\delta\mu}{4\,\pi}\,, \qquad \nu \cong (2\,\mu \mid \delta\mu \mid)^{1/2}\,, \tag{20}$$

and the value of the crucial parameter ν/ε is then

$$\frac{\nu}{\varepsilon} \cong \frac{g^2}{8\,\pi} \left(\frac{2 \mid \delta\mu \mid}{\mu}\right)^{1/2}. \tag{21}$$

For any given value of $\delta\mu/\mu$, (21) determines a critical value g_c of g which makes $\nu/\varepsilon = 1$:

$$g_c^2 = 4\,\pi \left(\frac{2\,\mu}{\mid \delta\mu \mid}\right)^{1/2}. \tag{22}$$

(E.g. for $\mid \delta\mu \mid = \mu/10$: $g_c = 7.5$.) Letting g (which must be $\gg 1$ anyway for our approximation to be valid) increase through the value g_c, the charge-symmetric spectrum is drastically changed, approaching either (18) or (19). It is worth noting that this need not require very large g values provided $\mid \delta\mu \mid/\mu$ is not too small.

3. Nucleon and Pseudoscalar Mesons

Instead of (3), the interaction is now

$$H' = g \sum\nolimits_{\varrho i} \tau_\varrho\, \sigma_i\, q_{\varrho i}\,, \quad q_{\varrho i} = \int d^3x\; v(x)\, \frac{\partial \varphi_\varrho(x)}{\partial x_i}\,. \tag{1a}$$

With the gradient coupling, g now has the dimension of a length, and the strong-coupling condition is $g \gg a$, where a = source radius $[\int d^3x\, v^2(x) \sim a^{-3}]$; a must be kept non-zero to avoid divergences, but we shall assume $a \ll \mu_\varrho^{-1}$.

How to diagonalize the 4×4 matrix H' is well known[7]). Most relevant is the lowest eigenvalue:

$$-g\,(r_1 + r_2 + r_3) \tag{4a}$$

where the r_n are the positive square-roots of the eigenvalues of the tensor $T_{\varrho\sigma} = \Sigma_i\, q_{\varrho i}\, q_{\sigma i}$. The corresponding eigenvectors will be called $s_{\varrho n}$ $[\Sigma_\varrho\, s_{\varrho n}\, s_{\varrho m} = \delta_{nm}]$. The expressions (6) and (7) can still be used if r is replaced by $\Sigma_n\, r_n$, and the Equations (9), (10), (11) have to be changed as follows:

$$Y_\varrho = \frac{1}{3}\int d^3x\, v(x)\,(\mu_\varrho^2 - \Delta)^{-1}\,(-\Delta)\, v(x)\,, \tag{9a}$$

$$r_n^0 = g\,\left(\sum\nolimits_\varrho s_{\varrho n}^2\, Y_\varrho^{-1}\right)^{-1}, \tag{10a}$$

$$V^0 = -\frac{1}{2}\, g \sum\nolimits_n r_n^0 = -\frac{1}{2}\, g^2 \sum\nolimits_n \left(\sum\nolimits_\varrho s_{\varrho n}^2\, Y_\varrho^{-1}\right)^{-1}. \tag{11a}$$

In the charge-symmetric limit ($r_n^0 = g\,Y$), H_{rot} involves now 3 Eulerian angles ϑ, φ, ψ [H_{rot} = energy of a spherical top; for $-i\,\partial/\partial\psi \equiv J_z = {}^1/_2$ it reduces to (13)], the eigenvalues are again given by (16), but now I stands for both the ordinary and the isotopic spin ($J = I$), and both projections J_z and I_3 run independently from $-I$ to $+I$ (degree of degeneracy $= (2I+1)^2$ if $\mu_\varrho = \mu$). The value of ε in (16) is changed as follows:

$$\varepsilon = 3\, g^{-2}\left[\int d^3x\, v(x)\,(\mu^2 - \Delta)^{-2}\,(-\Delta)\, v(x)\right]^{-1}. \tag{14a}$$

Allowing $\mu_1 = \mu_2 \neq \mu_3$, while destroying the isotropy in charge space, we retain, of course, the isotropy in the x_i space; the rotational energy will become dependent on $|I_3|$ but not on $|J_z|$ ($z \equiv x_3$). Since J_z and I_3 are projections of the same "angular momentum" on two different axes (analogous to "space-fixed" and "body-fixed"), one might anticipate that the I_3-dependence of the energy is somewhat inhibited, compared with the scalar theory (where $J = {}^1/_2$: scalar mesons are bound in S-states). That such is indeed the case,* follows from (11a), as compared with (11):

Let us write $Y_\varrho = Y\,(1 + \delta_\varrho)$, where $\Sigma_\varrho\, \delta_\varrho = 0$, and expand r_n^0 (10a) in powers of δ_ϱ:

$$r_n^0 = g\,Y\,[1 + \sum\nolimits_\varrho \delta_\varrho\, s_{\varrho n}^2 - \sum\nolimits_\varrho \delta_\varrho^2\, s_{\varrho n}^2 + (\sum\nolimits_\varrho \delta_\varrho\, s_{\varrho n}^2)^2 + \ldots]\,.$$

Substituting this in (11a) and carrying out the n-summation, we can use the completeness relation $\Sigma_n\, s_{\varrho n}^2 = 1$ which makes the *terms linear in δ_ϱ vanish identically*[8]). But these are precisely the terms which in the scalar theory ($\Sigma_n\, s_{\varrho n}^2 \to e_\varrho^2$) gave rise to the large effects we discussed. Instead of these terms ($\sim |\delta_1 - \delta_3|$), we now have a much reduced anisotropy of V^0 ($\sim |\delta_1 - \delta_3|^2$), and of a more complicated kind. Moreover, according to (9a) (assuming $a\,\mu_\varrho \ll 1$) δ_ϱ has the order $a^2\,\delta(\mu^2)$ [as against

*) *Note added in proof:* More generally, this is true even if a primary symmetry violation is admitted in the coupling constants also, as has been shown by R. Ramachandran.

$a\ \delta\mu$ in the scalar theory]. We find for the order of magnitude of the coupling strength at which a *strong* violation of charge-symmetry sets in:

$$g_c^2 \gtrsim |\mu_1^2 - \mu_3^2|^{-1} . \tag{22a}$$

This implies a very large value for the dimensionless coupling constant g_c/a if $|\delta\mu| \ll \mu$, as would be the case for real π-mesons.

4. Octet Model

We cannot discuss broken $SU(3)$ symmetries in the same generality because even the symmetric theories are insufficiently known at this time. We concentrate on a special case which is comparatively simple and also similar enough to the scalar meson-nucleon problem (section 2) to make a condensed review possible: this is the (almost) $SU(3)$ symmetric *scalar* theory with *pure D coupling*[9]).

Here we can start again from the Hamiltonian (1), (2), (3), where, however, now the index ϱ runs from 1 to 8, referring to an octet of scalar mesons, viz., 1, 2, 3 to an isovector "π", 4 ... 7 to two isospinors "K", 8 to an isoscalar "η". $\tau_1, \ldots, \tau_8$ are 8×8 matrices; τ_ϱ describes the transitions between bare baryon states (N, Λ, Σ, Ξ) which accompany the emission or absorption of a meson q_ϱ[10]). Hence H' is now an 8×8 matrix depending on the (real) variables $q_1, \ldots, q_8$. Its lowest eigenvalue is[9])[11])

$$-\,g\,r, \text{ where } r = \left(\sum\nolimits_\varrho q_\varrho^2\right)^{1/2} , \tag{4b}$$

characteristically spherically symmetric in the 8-dimensional q_ϱ-space. Two comments, however, must be added here. Firstly, this spherical symmetry obtains only for pure D-coupling (DE SWART's $\alpha_p = 0$); the slightest F admixture would destroy it. Secondly, the next higher eigenvalue of H' becomes degenerate with (4b) in certain directions q_ϱ/r, so that the corresponding potential valleys overlap in a small region in 8-space; the overlap of the corresponding wave-functions is, however, insufficient to affect the large ($\sim g^2$) energy terms we consider here, so we will ignore this complication.

Then, the "potential" V is again given by (7), and its minimum by (8), (9), (10), and (11), where now $e_\varrho = q_\varrho/r$ denotes a unit vector in 8-space. In the case of complete $SU(3)$ symmetry (all $\mu_\varrho = \mu$, all $Y_\varrho = Y$, $r^0 = g\,Y$) the valley is spherical in the q_ϱ-space, and even though the U transformation in (6) will introduce non-spherical terms, the "rotational" spectrum will display at least the $SU(3)$ symmetry of the model (irreducible representations 1, 8, 27, ...). We are here concerned with the question how this highly symmetric spectrum is altered if we allow the meson masses μ_ϱ to violate the $SU(3)$ symmetry (but not the charge symmetry).

Let us first choose a situation similar to "reality":

$$\mu_1 = \mu_2 = \mu_3 = \mu_\pi < \mu_K \sim \mu_\eta$$

or

$$Y_1 = Y_2 = Y_3 = Y_\pi > Y_K \sim Y_\eta \tag{23}$$

[see (20)]. The lowest value of the potential (11), viz. $V^0 = -\,{}^1/_2\, g^2\, Y_\pi$, is then reached for

$$e_1^2 + e_2^2 + e_3^2 = 1,\ e_4 = \ldots = e_8 = 0\,. \tag{24}$$

If the coupling is strong enough, viz. $g > g_c$ [for the order of magnitude of g_c, see (22)], the wave-function shrinks into the vicinity of the region (24), and to derive the rotational spectrum analogous to (18) [$\nu/\varepsilon \to \infty$, see (21)], we can neglect $e_4, \ldots, e_8$ (i.e. K and η mesons) altogether. This has the simplifying consequence that the 3 bare-baryon groups of different hypercharge (N, $\Lambda + \Sigma$, Ξ) are decoupled (with $K \to 0$, the matrix H' reduces to one 4×4 and two 2×2 matrices), and it is easily seen[10]) that the lowest eigenvalue (4b) of H' belongs to the 4×4 submatrix involving transitions between the four (bare) Λ and Σ states only. In other words, the bare N and Ξ can be ignored also. The remaining problem (interaction "$\Lambda \Sigma \pi$" only) has already been solved in the strong-coupling approximation[12]). The energy spectrum has again the rotational structure (16), but with integral isospin: $I = 0$ ("Λ"), 1 ("Σ"), 2, ... Nothing remains (for $g > g_c$) of the $SU(3)$ symmetry, and again, it is very "easy" to break the symmetry if $|\delta\mu|/\mu$ is not too small [see (22)].

We have also studied the case that the "K-meson" is lighter than both "π and η" so that (24) is replaced by

$$e_4^2 + e_5^2 + e_6^2 + e_7^2 = 1, \quad e_1 = e_2 = e_3 = e_8 = 0 \,. \tag{25}$$

Then, of course, there is no decoupling between any of the bare baryon states, and the problem is much more involved. We have derived, for $\nu/\varepsilon \to \infty$, the rotational Schrödinger equation in suitable polar coordinates[13]). (All bare baryons are mixed in accordance with their statistical weights.) The lowest stationary state is a physical "Λ" ($I = 0$), next higher are "N" and "Ξ" ($I = 1/2$), whereas the "Σ" ($I = 1$) follows in the second higher group which also contains two isotriplets of hypercharges ± 2.

Particularly simple is the case $\mu_\eta < \mu_\pi$ and μ_K ($e_8 = \pm 1$). Then, each baryon is eventually ($g \gg g_c$) coupled only to itself, the negative self-energies are largest (namely $\frac{1}{2} g^2 Y_\eta$) for the Λ and Σ, so the spectrum, asymptotically, consists only of a "Λ" and a "Σ", degenerate.

In all this, we assumed scalar mesons and pure D coupling. For pure F (or mixed) coupling, analogous results could presumably be derived, despite the fact that complete $SU(3)$ symmetry does no longer imply spherical symmetry in the 8-dimensional q_ϱ space[14]). A more interesting question is what happens if the scalar mesons are replaced with pseudoscalar ones. For the nucleon-pion system (sections 2 and 3) we noticed that the charge-symmetry of the rotational states is less easily broken if the mesons are pseudoscalar, owing to the interdependence of spin and isospin. We expect that a similar difference in sensitivity to symmetry-breaking causes will also appear in the octet model.

References

1) M. Gell-Mann, Phys. Rev. *125*, 1067 (1962).

2) S. Okubo, Prog. Theor. Phys. *27*, 949 (1962).

3) G. Wentzel, Helv. Phys. Acta *13*, 269 (1940) and *14*, 633 (1941). Subsequent work is quoted in G. Wentzel, Rev. Mod. Phys. *19*, 1 (1947), ref. 24.

4) Otherwise $g\, q_\varrho \to g_\varrho\, q_\varrho$. This generalization could easily be carried through, but would not introduce interesting new features in the results.

5) Small higher-order effects have been discussed (in the pseudoscalar theory) by A. HOURIET, Helv. Phys. Acta *18*, 473 (1945).

6) More precisely: $\cos\vartheta = (-)^{n+1} I_3/|I_3|$.

7) W. PAULI and S. M. DANCOFF, Phys. Rev. *62*, 85 (1942), Section 4; G. WENTZEL, Helv. Phys. Acta *16*, 551 (1943), § 4.

8) For $|\delta\mu| \ll \mu$, this is most easily seen by going back to the original Hamiltonian (1): Write in (2)

$$\mu_1^2 = \mu_2^2 = \mu^2 + \alpha, \; \mu_3^2 = \mu^2 - 2\alpha,$$

and treat the term $\sim\alpha$ as a small perturbation.

9) G. WENTZEL, EFINS report 64-33 (unpublished).

10) The general formula for the $SU(3)$-symmetric Yukawa interaction, as first published by S. OKUBO[2]) [see his equations (21)], defines the matrices τ_ϱ. See also J. J. DE SWART, Rev. Mod. Phys. *35*, 916 (1963), equations (17.7 and 8), where $\alpha_p = 0$ corresponds to pure D coupling.

11) Here, g stands for the coupling constant $|g_{\Lambda\Sigma\pi}|$, i.e. in DE SWART's[10]) notation: $g = {}^2/_3 \sqrt{3}\, |g_p|$ $(\alpha_p = 0)$.

12) G. WENTZEL, Phys. Rev. *125*, 771 (1962). The treatment there is more general in that both $\Lambda\Sigma\pi$ and $\Sigma\Sigma\pi$ couplings are admitted. For present purposes (pure D), the second coupling constant (g') vanishes, so that only 'case I' is relevant [see equations (12), (13), (21), (22)]. The simplifying assumption of 'low cutoff' ($a\mu \gg 1$) affects essentially only the coefficient of L^2 in (17).

13) For their definition, see G. WENTZEL, Helv. Phys. Acta *30*, 135 (1957), equations (1) to (4). However, the U transform in T (6) introduces additional terms of an unfamiliar kind.

14) For instance, for pure F coupling [$SU(3)$-symmetric, all $\mu_\varrho = \mu$] the locus of the V-valley is the intersection of an 8-sphere with the invariant surface

$$\Sigma_{\varrho\sigma\tau}\, d_{\varrho\sigma\tau}\, q_\varrho\, q_\sigma\, q_\tau = 0\,,$$

where d = GELL-MANN's symmetric tensor [see ref. 1, eq. (4.11) and table II][9]). One remarkable rotational excitation is an «icosuplet», corresponding to the irreducible representations {10} and {10*} of the $SU(3)$ group, degenerate.