Zeitschrift: Helvetica Physica Acta

Band: 38 (1965)

Heft: III

Artikel: Symmetry breaking two-body photonic decay of vector mesons and unitary symmetry

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DOI: https://doi.org/10.5169/seals-113593

Symmetry Breaking Two-Body Photonic Decay of Vector Mesons and Unitary Symmetry*)

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(23. XII. 64)

Abstract: Two-body photonic decay of vector mesons is investigated in the symmetry-breaking unitary-symmetry model of GELL-MANN and NE'EMAN. We derive three equations from nine decay amplitudes under charge conjugation invariance. The above relations will allow us to determine the ω-ϕ mixing angle when enough experimental information is available. A method to study this problem is suggested.

I. Introduction

Two-body photonic decay of the known vector mesons is to be investigated by the unitary-symmetry model of GELL-MANN[1]) and NE'EMAN[2]). There are several papers[3-5]) on this problem but none of them considered the effect of symmetry breaking. The symmetry breaking due to the medium-strong interaction (which transforms as an isosinglet and a hypercharge zero member of the octet) for the current operator was discussed in a previous paper[6]). In our present approach there arise six independent parameters under charge conjugation invariance and nine amplitudes, so we get three relations which will be compared with the experimental data. In the actual problem an amplitude for $K^{*+} \to K^+\gamma$ decay remains uncoupled with the other ones.

It is very important to consider the symmetry breaking when discussing the $\omega - \phi$ mixing angle[7]). The result obtained in this paper is exact and may be used for this purpose when we get enough experimental information.

Since the space-time structure of the above problem is known, the invariant phase volume which allows us to separate the absolute value of the amplitude derived by the $SU(3)$ group postulate can be given. At this moment the problem solved here is one of the few studies which include the effect of the symmetry breaking on the vectormeson decays[8]). Therefore it is very important to study the decay rate of the suggested rare decay modes experimentally. We hope that our approach gives a useful tool for determining the $\omega - \phi$ mixing angle.

In Section II we shall derive the two-body photonic decay amplitude of vector mesons. In Section III three relations between amplitudes will be given together with

*) This work is supported by the Swiss National Science Foundation.

the invariant phase volume. In the last section we shall discuss our result in connection with the available experimental information.

II. Photonic Decay Amplitudes for Vector Mesons

The electromagnetic current E (1) in the first order of the symmetry breaking medium-strong interaction[6]) is, in de SWART's notation[9]), equal to

$$\left.\begin{aligned} E(1) = {} & T^{(8)}_{0,0,0} + \frac{3}{2} V^{(27)}_{0,0,0} - V^{(8_1)}_{0,0,0} + V^{(1)}_{0,0,0} \\ & + \sqrt{3}\,(T^{(8)}_{1,0,0} + V^{(27)}_{1,0,0} + V^{(8_1)}_{1,0,0}) + V^{(10)}_{1,0,0} + V^{(10^*)}_{1,0,0}\,, \end{aligned}\right\} \quad (1)$$

where T and V are the symmetry nonbreaking and the breaking current respectively Let us define the $\omega - \phi$ mixing angle θ by

$$\left.\begin{aligned} \phi &= \phi^0 \cos\theta - \omega^0 \sin\theta \\ \omega &= \phi^0 \sin\theta + \omega^0 \cos\theta\,, \end{aligned}\right\} \quad (2)$$

where ϕ and ω are the corresponding vector meson fields and those with zero superscript are the ones for the octet and the singlet member of the irreducible representation of the $SU(3)$ group. Calculation of the decay amplitude is straightforward[9]). Defining the reduced matrix elements

$$a = \frac{\sqrt{5}}{5}\,(8 \parallel T^{(8)} \parallel 8)_1\,, \quad b = \frac{\sqrt{5}}{30}\,(8 \parallel V^{(27)} \parallel 8)\,, \quad c = \frac{\sqrt{5}}{5}\,(8 \parallel V^{(8_1)} \parallel 8)\,,$$

$$d = (8 \parallel V^{(1)} \parallel 8)\,, \quad e = (8 \parallel T^{(8)} \parallel 1) \quad \text{and} \quad f = (8 \parallel V^{(8_1)} \parallel 8)_1$$

(all other reduced matrix elements are zero under the charge conjugation invariance) we obtain the two-body photonic decay amplitudes of the vector mesons tabulated in Table 1. The parameters a, b, c, d and e, f in the Table must be multiplied by $\sin\theta$ and $\cos\theta$ for the ω decay and $\cos\theta$ and $-\sin\theta$ for the ϕ decay respectively. The symmetry-nonbreaking solution of TANAKA[5]) is, of course, obtained by equating $b = c = d = f = 0$.

III. Relation between Amplitudes

There are nine decay modes and six independent form factors (see Table 1). We get three relations between amplitudes A's, i.e.,

$$A(\varrho^+ \to \pi^+ \gamma) = A(\varrho^0 \to \pi^0 \gamma)\,, \quad (3)$$

$$\sin\theta\, A(\omega \to \pi^0 \gamma) + \cos\theta\, A(\phi \to \pi^0 \gamma) = A(\varrho^0 \to \eta\, \gamma) \quad (4)$$

and

$$\left.\begin{aligned} & 3 \sin\theta\, A(\omega \to \eta\, \gamma) + 3 \cos\theta\, A(\phi \to \eta\, \gamma) + A(\varrho^+ \to \pi^+ \gamma) \\ & \qquad - 2\sqrt{3}\, A(\varrho^0 \to \eta\, \gamma) = 4\, A(K^{*0} \to K^0 \gamma)\,. \end{aligned}\right\} \quad (5)$$

Note that $A(K^{*+} \to K^+ \gamma)$ remains uncoupled with the other amplitudes. These equations contain the solution obtained from the symmetry nonbreaking as a special case (see reference [5])).

Table 1
Two-body photonic decay amplitude for vector mesons

Process	coefficients of a	b	c	d	e	f
$\varrho^+ \to \pi^+ \gamma$	1	-1	-1	1	—	—
$\varrho^0 \to \pi^0 \gamma$	1	-1	-1	1	—	—
$\varrho^0 \to \eta \gamma$	$\sqrt{3}$	$-4\sqrt{3}$	$\sqrt{3}$	—	—	—
$\omega \to \pi^0 \gamma$	$\sqrt{3}$	$-4\sqrt{3}$	$\sqrt{3}$	—	$\sqrt{3}$	$\sqrt{3}$
$\omega \to \eta \gamma$	-1	-9	1	1	1	-1
$\phi \to \pi^0 \gamma$	$\sqrt{3}$	$-4\sqrt{3}$	$\sqrt{3}$	—	$\sqrt{3}$	$\sqrt{3}$
$\phi \to \eta \gamma$	-1	-9	1	1	1	-1
$K^{*+} \to K^+ \gamma$	1	7	2	1	—	—
$K^{*0} \to K^0 \gamma$	-2	-1	-1	1	—	—

The invariant R-matrix element in momentum space is given by $e_{\alpha\beta\gamma\delta}\, Q_\alpha\, \varepsilon_\beta\, k_\gamma\, \lambda_\delta$ where $e_{\alpha\beta\gamma\delta}$ is a completely antisymmetric Levi-Civita tensor, ε and λ are the polarization vectors of the vector meson and the photon respectively and Q and k are the corresponding four-momenta. Making use of the above expression the invariant phase volume (in the rest system of the vector meson) is equal to $(M^2 - m^2)^3/M^2$ where M and m are the masses of the vector meson and the pseudoscalar meson respectively. The partial decay rate W for each decay is expressed as

$$W = \text{const.}\, \frac{(M^2 - m^2)^3}{M^3} | A |^2 , \tag{6}$$

where the constant is common for all the decay rates discussed in this paper. We can estimate the absolute value of A from the experimental data by making use of Equation (6). Generally speaking we shall obtain only the information for the absolute size of A which will be inserted in Equations (3, 5) after reducing them to relations between absolute values. We shall get a triangular relation and a multiangular relation (a closed polygon) for the complex functions A's in the momentum space; 'inequalities' for the absolute value of the amplitudes. However, if the amplitudes are real within a sufficient accuracy*) we can determine the $\omega - \phi$ mixing angle directly from Equations (4) and (5), otherwise they will serve as a tool for an inconsistency check.

IV. Discussion

The relations obtained in this paper are exact and can be applied to the determination of the $\omega - \phi$ mixing angle, or for the inconsistency check.

*) This condition is satisfied if, for instance, the pole approximation model holds.

Unfortunately experimental information[10,11] is too poor to estimate the above angle. There is some information for $\omega \to \pi^0 \gamma$ decay rate[11]. It is necessary to measure the $\phi \to \pi^0 \gamma$ and $\varrho^0 \to \eta \gamma$ decay rates in order to use Equation (4). The latter decay will be masked by $\pi^0 \gamma$ mode and/or suffer from the difficulties associated with the measurement to distinguish between ϱ^0 and ω[10]. We should like to point out how to solve this question experimentally. In the symmetry nonbreaking case[5] we have $A(\varrho^0 \to \eta \gamma) = \sqrt{3}\, A(\varrho^0 \to \pi^0 \gamma) = \sqrt{3}\, A(\varrho^+ \to \pi^+ \gamma)$ (a second equality which is exact; see, Equation (3)). According to reference [10] the relative decay rate of the rare decay mode of ϱ varies between $10^{-2} - 10^{-5}$. If the $\varrho^+ \to \pi^+ \gamma$ decay rate turns out to be very small we can neglect $A(\varrho^0 \to \eta \gamma)$ in Equation (4) in virtue of the above relation*). If this is really the case a single measurement on $\phi \to \pi^0 \gamma$ decay rate will be sufficient for determining the $\omega - \phi$ mixing angle. A detailed discussion of this type is a problem for experimental physicists. For a theoretician a road is open to another investigation[8].

We hope that our approach eventually gives a consistent and a reliable answer to all the related questions.

Acknowledgement

The author expresses his cordial thanks to Professor A. MERCIER for his interest in this paper, for the careful reading of the manuscript and for his kind hospitality.

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*) Notice that $(M^2 - m^2)^3/M^3 = 0.044$ and 0.389 $(GeV)^3$ for $\varrho^0 \to \eta \gamma$ and $\varrho^0 \to \pi^0 \gamma$ decay respectively.