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The T(d, n)^{4}He Reaction at Low Energies

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Abstract. A crude model is given of the mechanism of the $d+t \rightarrow \alpha+n$ reaction, for $E_d =$ 500 KeV, which includes the effect of the resonance, observed at $E_d = 107$ KeV, together with a contribution from a stripping mode. Comparison is made with the observed differential cross sections and the angular distributions associated with a polarized deuteron beam where the latter is described by different values of P_{33}, $P_{11} - P_{22}$, and P_2.

1. Introduction

The resonance reaction, $d + t \rightarrow \alpha + n$ presents a versatile tool for nuclear studies. With the large cross section at the resonance energy, $E_d = 109$ KeV, we are assured of a copious source of neutrons in the 14 MeV energy range. Further, the resonance state is one of the few examples of an extended, but isolated, deuteron resonance; indeed there is no evidence of any higher resonance state of the ^{5}He system. The remarkable simplicity of the process, as currently depicted, in fact, accounts for its unique role as a polarimeter for low-energy deuterons.

The apparent absence of other resonance states is of some interest. One might even reject the usual view of a resonance at 109 KeV, and classify the process as some threshold phenomenon in the entrance channel, as has been done by BAZ[1]). Alternatively, the resonance might be accepted and the lack of observation of other resonances might be attributed to distant, broad overlapping states. A precise description in these terms is impossible, although some attempt has been made by BREIT and collaborators[2]) to account for these states. PEARLSTEIN, TANG, and WILDERMUTH[3]) have pointed to the large intrinsic deuteron-triton width which is derived from such an analysis as evidence of strong deuteron-triton clustering in a relative s state. This is in contrast to the characteristic $\alpha - n$ clustering of the two lower states of ^{5}He.

A more detailed description of the compound system, inevitably requires a characterization in terms of the individual five nucleons and the formalism becomes greatly complicated. Some light might be shed, at least on the spin-dependent configurations, by further experimental study, utilizing the full potentiality of a polarized deuteron beam. With sufficiently large polarization tensors, measurement of the associated efficiency tensors at energies somewhat removed from the resonance energy would

lead to a specification of certain terms of the reaction matrix. The resultant information about channel-spin mixtures would then relate to the type of spin-dependence manifested by the individual nucleon-nucleon interactions.

The Basel experiment[4]) represents a first step in this direction. One of the results of this analysis points to the relatively small contributions associated with the non-resonant terms*) of the reaction matrix, thus making difficult any detailed determination of the various matrix elements. The number of elements increase sharply with orbital angular momentum. Even, if restriction is given to deuteron s and p waves only and if f waves are ignored in the exit channel there arise twelve complex parameters for each energy and angle, and a study of the angular dependence of the efficiency tensors $\varepsilon_{k\varkappa}$ would have to be supplemented by information deducible from measurement of the neutron polarization with unpolarized deuterons. Even if allowance is given to very small experimental errors, the determination of the individual matrix elements would seem to be precluded.

The experimental results of BAME and PERRY[5]) which deal with differential cross sections for various energies from $E_d = 500$ KeV up to several MeV, already reveal the effect of non-resonance contributions. With the sharp parity (+) of the resonance state, the angular distribution should be symmetric about 90° and even isotropic, except for possible small contributions from deuteron d waves. Actually, terms appear in the angular distribution which vary as $P_1(\cos\theta)$ and point to contributions from negative-parity states of the compound system.

Further information about the reaction is obtained at higher energies. The angular distribution, found by BROLLEY et al.[6]) for $E_0 = 10.5$ MeV, was cited by BUTLER and SYMONDS[7]) as evidence in support of the stripping mechanism. Here, a proton is transfered with relative orbital angular momentum, $l_p = 0$. The mirror reaction, $He^3(d, p)$ has also been studied from this viewpoint and the angular distribution has been fitted[8]) using the distorted wave BORN approximation (D.W.B.A.) in spite of the difficulty in depicting the deuteron distortion.

We propose to reexamine the reaction at $E_d = 500$ KeV, by adding to the resonance contribution, a stripping amplitude, which is to be treated using the D.W.B.A. The Coulomb barrier is large enough ($\sim$ 1 MeV) to inhibit close approach of the deuteron because of the relatively large deuteron size and the small charge of the target. If, in fact, this inhibition were significant, it would have influenced the resonance cross section as well. The unusually large binding energy restricts the stripping to grazing of the triton by the deuteron or even to mutual penetration. Consequently only a few partial-waves are significant. The stripping angular distribution, by itself, does not deviate too much from isotropy since mainly s, p, and d waves enter; yet it is the interference between the s and p waves which accounts for the $P_1(\cos\theta)$ terms observed in the angular distribution.

Our view of the reaction is admittedly a crude one whose virtue rests in the comparatively few parameters that are added to the amplitude. If comparison is made to other treatments using the R^∞ term, the present procedure may be interpreted as providing a reasonable model for this term.

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*) The "non-resonant" terms of the reaction matrix refer to those terms which are additional to those associated with the resonance at $E_d = 109$ KeV.

2. Pertinent Formulae

Our notation follows that used in ref. 9. Referring to an arbitrary binary process, the spins of the target and residual nuclei are represented by a and c with components α and γ, respectively, while the angular momentum of the intermediate state is specified by b and β. Channel spins, S_1 and S_2, arise in the coupling of a to the projectile spin, s_1, and c to the spin of the reaction product, s_2. Further coupling of S_1 to l_1 and S_2 to l_2 yields b. Representing the polarization of the projectile by $\varrho_{k_s\varkappa}(s_1)$, the angular distribution is found to be [9]:

$$\frac{d\sigma}{d\Omega} = \frac{\hat{\lambda}^2}{4\,\hat{a}^2\,\hat{s}_1^2} \sum (-)^{a - s_1 + k_s + l_1' + S_2 - s_1' - b'} \hat{k}_s\,\hat{k}_l\,\hat{S}_1\,\hat{S}_1'\,\hat{l}_1\,\hat{l}_1'\,\hat{l}_2\,\hat{l}_2'\,\hat{b}^2\,\hat{b}'^2\,\hat{s}_1\,\varrho_{00}^{-1}(s_1)$$

$$(l_1\,0, l_1'\,0 \mid k_l\,0)\,(l_2\,0, l_2'\,0 \mid k0)\,W(s_1\,s_1\,S_1\,S_1'; k_s\,a)\,W(l_2\,b\,l_2'\,b'; S_2\,k)$$

$$\begin{Bmatrix} S_1 & l_1 & b \\ S_1' & l_1' & b' \\ k_s & k_l & k \end{Bmatrix} B_1(k_s\,k_l\,k;\ R\,R'^*)\,,$$

where

$$B_1(k_s\,k_l\,k;\ R\,R'^*) = \varrho_{k_s0}(s)\,(k_s\,0, k_l\,0 \mid k\,0)\,P_k(\cos\theta)\mathrm{Re}(R\,R'^*)$$

$$+ 2\sum_{\varkappa>0}{}'(-)^{\varkappa}\sqrt{\frac{(k-\varkappa)!}{(k+\varkappa)!}}\,(k_s\,\varkappa, k_l\,0 \mid k\,\varkappa)\,P_k^{\varkappa}(\cos\theta) \begin{cases} \mathrm{Re}\left(\varrho_{k_s\varkappa}(s)\right)\mathrm{Re}\,(R\,R'^*) \ldots \text{even } k_s \\ -\mathrm{Im}\left(\varrho_{k_s\varkappa}(s)\right)\mathrm{Im}\,(R\,R'^*) \ldots \text{odd } k_s \end{cases}$$

Here, R, stands for elements $\langle (c\,s_2)\,S_2, l_2, b \mid R \mid (a\,s_1)\,S_1, l_1, b\rangle$, $\hat{x}$ represents $(2x+1)^{1/2}$ and $\hat{\lambda}$ is the reduced incident wave number in the centre-of-mass system. The spin tensors are defined so that the z-axis is along the direction of motion of the incoming beam, while the y-axis is along the normal to the reaction plane, $\boldsymbol{k}_i \times \boldsymbol{k}_f$.

Great simplification is obtained if we deal with the $d + t$ reaction, for then, $c = 0$, $a = s_2 = S_2 = 1/2$ and $s_1 = 1$. The quantum numbers, appropriate to the resonance reaction, include $l_1 = 0$ and 2, $b = S_1 = 3/2$. According to the BREIT-WIGNER formalism [10], the corresponding reaction matrix element is of the form

$$\langle S_2\,l_2\,b \mid R_{res} \mid S_1\,l_1\,b\rangle = i\,e^{i(\xi_{l_1} + \xi_{l_2})}\,\frac{\Gamma^{1/2}(x_1\,S_1\,l_1)\,\Gamma^{1/2}(x_2\,S_2\,l_2)}{E_0 - E - i\,\Gamma/2}$$

where

$$\Gamma = \sum_{x\,S\,l}\Gamma(x\,S\,l) = 2\sum_{x\,S\,l}{}'P_l(x)\,\gamma_x^2(S\,l)\,,$$

the channel index being labelled x. The partial width $\Gamma(x\,s\,l)$ is related to the channel reduced width $\gamma_x^2(S\,l)$ though the penetration factors $P_l(x)$. The quantities, ξ_l represent the sum of a Coulomb phase shift, σ_l, and a hard-sphere phase shift, ϕ_l. The resonance occurs at $E = E_0 = E_\lambda + \Delta_\lambda$ where E_λ is the formal resonance energy, and Δ_λ is the level shift, which, for this specific reaction, is sizeable [2]. Usually, R_{res} is multiplied by $\exp[-i\{\sigma_0(1) + \sigma_0(2)\}]$, so that the combination $\exp[i(\sigma_l - \sigma_0)]$ enters, which is relatively simple. The direct-reaction amplitude, on the other hand, is normally calculated numerically with the asymptotic behaviour of the distorted waves deter-

mined by σ_l, among other factors. Since σ_l must therefore be determined, there is no point to introducing the σ_0 factor, except when necessary.

In the presence of a direct-reaction contribution, the reaction matrix is written as $R = R_{res} + R_D$ where R_D is given by THOMAS[11]) for the case of spin-dependent distortions and by GOLDFARB and JOHNSON[12]) for the more general case. Limiting ourselves to stripping associated with an s-wave nucleon, we have

$$\langle (c\, s_2)\; S_2, l_2, b \mid R_D \mid (a\, s_1)\; S_1, l_1, b \rangle = \delta_{S_1 S_2}\, \delta_{l_1 l_2}\, \theta_{1/2}$$

$$\Psi_i \frac{\sqrt{m_1^* m_2^*}}{4\pi \hbar^4}\, k_i\, k_f\, A_{l_1}\, \hat{s}_1\, \hat{c} (-)^{S_1 - c - s_1 + 1}\, W(a\, s_1\, c\, s_2;\, S_1\, s)$$

where

$$A_l = \int f_l^{(-)*}(k_f, r)\, u_p(r)\, f_l^{(+)}(k_i, r)\, r^2\, dr\,.$$

The distorted-wave eigenfunctions are represented by $f_{\pm}^{l}(k, r)$ where the superscript indicates their asymptotic behaviour, while $u_p(r)$ refers to the proton eigenfunction. The distorted waves are normalized to be given by exp $(i\, \boldsymbol{k}\, \boldsymbol{r})$ in the absence of Coulomb and nuclear distortion. The quantity, $\theta_{1/2}$ represents the reduced-width amplitude for stripping while m_1^* and m_2^* are reduced masses in the entrance and exit channel, respectively. This result, which is essentially that of THOMAS[11]), is derived in the appendix where the case $l_p \neq 0$ is also considered.

An interesting consequence is the limitation of the stripping amplitude to $S_1 = 1/2$. With the characterization of the resonance by $S_1 = 3/2$ there should be no interference between R_{res} and R_D in the expression for the differential cross section. This merely follows as a result of the incoherence over S_1, which is associated with the fact that the deuterons are unpolarized. Any anisotropy about 90° is to be attributed to contributions from R_D alone.

The parameterization of the deuteron polarization is as follows[13])

$$\varrho_{00}(1) = \frac{1}{\sqrt{3}} \qquad \varrho_{20}(1) = \frac{1}{\sqrt{6}}\, P_{33}$$

$$\varrho_{10}(1) = \frac{1}{\sqrt{2}}\, P_{33} \qquad \varrho_{2\pm 1}(1) = \mp\, (P_{13} \mp i\, P_{23})$$

$$\varrho_{1\pm 1}(1) = \mp\, \frac{1}{2}\, (P_1 \mp i\, P_2) \qquad \varrho_{2\pm 2}(1) = \frac{1}{6}\, (P_{11} - P_{22}) \mp i\, \frac{P_{12}}{2}\,.$$

The expression for the angular distribution shows that only P_{33}, P_{13}, $P_{11} - P_{22}$ and P_2 arise, and the form of the angular distribution is then given by[13])

$$W(\theta) = W_0(\theta) \left[1 + \frac{3}{2}\, P_2\, P_2^a(\theta) + \frac{1}{2}\, P_{33}\, P_{33}^a(\theta) + \frac{2}{3}\, P_{13}\, P_{13}^a(\theta) + \frac{1}{6}\, (P_{11} - P_{22})\, (P_{11}^a(\theta) - P_{22}^a(\theta))\right],$$

where W_0 characterizes the unpolarized angular distribution and P_2^a, P_{33}^a, P_{13}^a, and $P_{11}^a - P_{22}^a$ are parameters, measuring the efficiencies of detection of the corresponding polarization parameters. The bounds to the efficiency parameters are given by

$$|\, P_2^a \,| \leqslant 1\,,\ |\, P_{13}^a \,| \leqslant {}^3/_2\,,\ |\, P_{11}^a - P_{22}^a \,| \leqslant 3\,,\ -2 \leqslant P_{33}^a \leqslant +1\,.$$

3. Numerical Results

The three reaction matrix elements, represented as $R(s)$, $R(d_{1/2})$, and $R(d_{3/2})$ are each associated with $b = 3/2$, $l_2 = 2$, and $S_2 = 1/2$. The analysis of CONNER, BONNER, and SMITH[14]) at energies near the resonance leads to the parameters for $R(s)$: $E_\lambda = -464$ KeV, $\gamma_d^2 = 2000$ KeV, and $\gamma_n^2 = 56$ KeV with the channels radii both taken as 5.0×10^{-13} cm. The level shift corresponding to 500 KeV deuterons is found to be $\Delta_\lambda = 176$ KeV. Since the penetrability factors, the Coulomb phase shift and the hard sphere phase shift are known for the s-wave contributions, the only unknown factors are the real quantities $\gamma_d(1/2, 2)$ and $\gamma_d(3/2, 2)$. For convenience, we introduce the related dimensionless factors c_1 and c_3 defined by

$$c_1 \equiv \frac{\gamma_d\left(\frac{1}{2}, 2\right)}{\gamma_d\left(\frac{3}{2}, 0\right)}, \qquad c_3 = \frac{\gamma_d\left(\frac{3}{2}, 2\right)}{\gamma_d\left(\frac{3}{2}, 0\right)}.$$

The main difficulty in determining the stripping amplitude rests in the characterization of the deuteron distortion. Indeed, it is not even clear that a potential should be introduced involving only the relative distance between the centres-of-mass of the deuteron and triton. Since, however, the stripping contribution is assumed to provide only a small perturbation, we shall not be too critical about the distorting potentials. The potentials are assumed to be spin-independent and of the form:

$$V_i^{opt} = -(V_i + i W_i)\left\{1 + \exp\left(\frac{(r - R_i)}{a_i}\right)\right\}^{-1} - U_i \exp -\left(\frac{(r - R_i)}{b_i}\right)^2,$$

where i refers, separately, to the deuteron and neutron. The associated parameters are represented in table 1.

Table 1

Optical Model Parameters for $d + t \rightarrow \alpha + n$

i	V_i	W_i MeV	U_i	a_i	b_i Fermis	R_i
d	50	5	0	0.60	0	1.5 $A^{1/3}$
n	40	0	2.5	0.18	0.20	1.3 $A^{1/3}$

The neutron parameters are taken to be consistent with those found for protons at 14.5 MeV[15]). The spin-orbit potential included in the analysis of ref. [15]) was neglected in this treatment. In the absence of other information, the deuteron parameters were supposed to be similar to those found for interactions with heavier nuclei, and were adjusted to provide the best fit for the differential cross section as found by BAME and PERRY[5]). The proton wave function was appropriate to a Woods-Saxon potential with potential strength adjusted to fit the binding energy of the proton in ^{4}He.

The radial integrals, A_l, decrease sharply with l so that terms such as $|A_2|^2$ and $\mathrm{Re}(A_1 A_2^*)$ may be safely disregarded, and the $P_1(\cos\theta)$ term in the differential cross section is seen to arise practically only from the interference between the s- and p-waves of the R_d amplitude.

The best choice of parameters consistent with the experimental data of BAME and PERRY[5]) corresponds to $\theta_{1/2} = \pm 0.2$, $c_1 = \pm 0.68$, and $c_2 = 0.23$ and the resultant fit is shown in figure 1. Also shown are the fits associated with $\theta_{1/2} = \pm 0.15$ and 0, to indicate the sensitivity to this parameter. The latter case corresponds to contributions from the resonance state only and the curve is seen to be symmetric about $\theta = 90°$, as is expected.

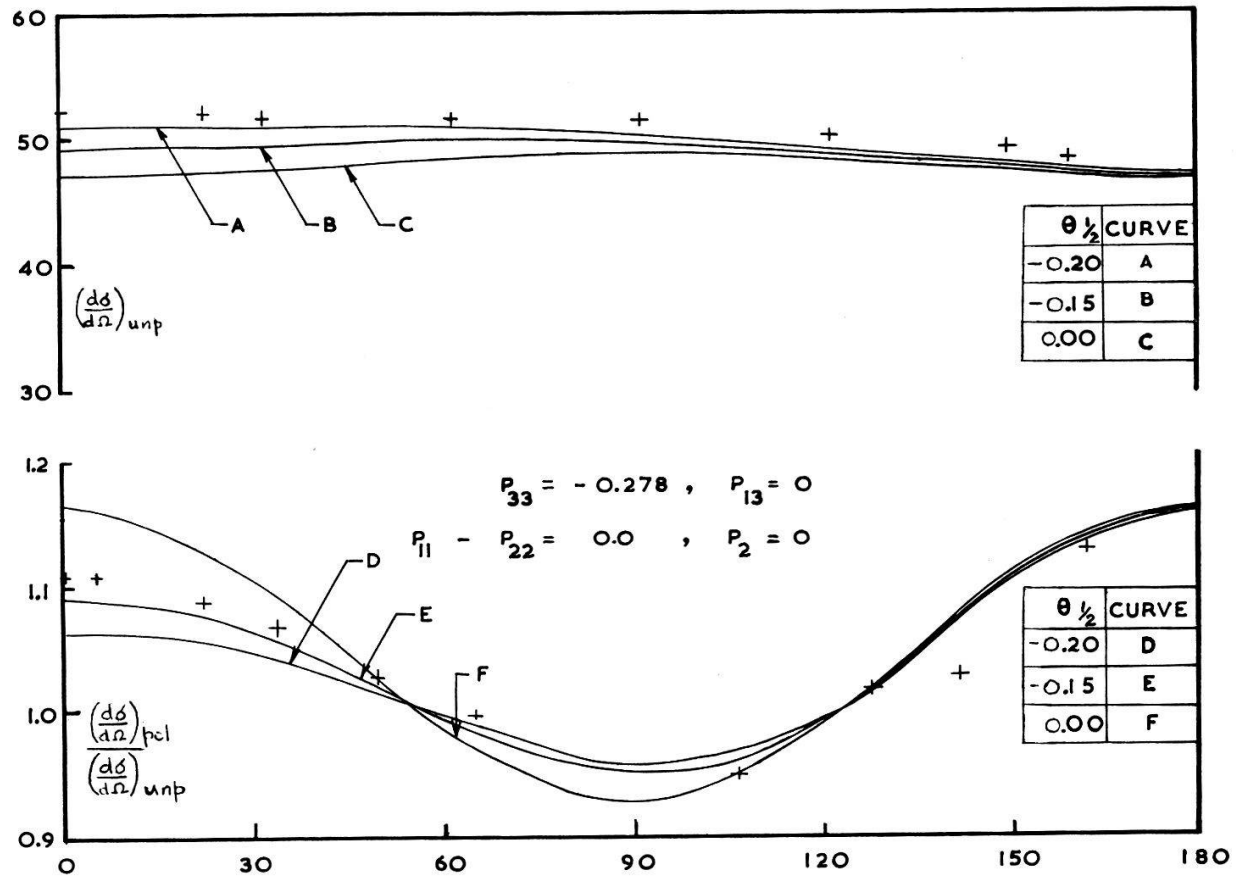

Figure 1

Unpolarized and polarized angular distributions for the $d+t$ reaction at $E_d = 500$ KeV, associated with contributions from the resonance and stripping mechanism, where $\theta_{1/2} = -0.15$, -0.20, and 0. The deuteron polarization is represented by $P_{33} = -0.278$. Comparison is made with the results of reference 4 and 5.

Also shown in figure 1 are the angular distributions associated with a 500 KeV polarized deuteron with $P_{33} = -0.278$, as compared with the experimental curve for $E_d = 490$ KeV. There are no contributions to P_{33}^a from R_d alone since the distortion of the deuterons was assumed to be spin-independent and the D.W.B.A. calculation, in this case, leads to results which are insensitive to the degree of tensor polarization[16]). Contributions to P_{33}^a only arise from the resonance matrix elements, in combination with each other or from the interference between $R(s)$ and R_D. There is now a sensitivity to the signs of $\theta_{1/2}$ and c_1 – best agreement being obtained for $\theta_{1/2} = -0.15$ and $c_1 = +0.68$.

Comparison may be made with the measurements[4]) corresponding to $E_d = 570$ KeV. The introduction of a magnetic field at the ionization region which is oriented with polar angle ϑ and azimuthal angle φ leads to a tensor $\bar{\varrho}_{ko}(1)$, referred to the

direction of the field. When referred to the coordinate system used for this reaction, there are generated tensors

$$\varrho_{k\varkappa}(1) = \bar{\varrho}_{k0}(1)\, D^k_{\varkappa 0}(\varphi, \vartheta, 0) = \bar{\varrho}_{k0}(1)_{(-)}^{\varkappa \delta_{\varkappa,|\varkappa|}} \sqrt{\frac{(k-|\varkappa|)!}{(k+|\varkappa|)!}}\, P_k^{|\varkappa|}(\cos\vartheta)\, e^{-i\varkappa\varphi},$$

Thus, if $\vartheta = \pi/2$, and $\theta = 0$ and π, the polarization is represented by $P_{33} = 0.139$ and $P_{11} - P_{22} = -0.417$, $P_{13} = P_{12} = 0$. Figure 2 shows a comparison of the results of experiments at 570 KeV with the predictions corresponding to the above parameters but at 500 KeV. Effects owing to the change in energy are not expected to be great. The comparison shows that a fairly reasonable fit is obtained from the resonance contributions alone and the stripping contribution fails to improve the situation.

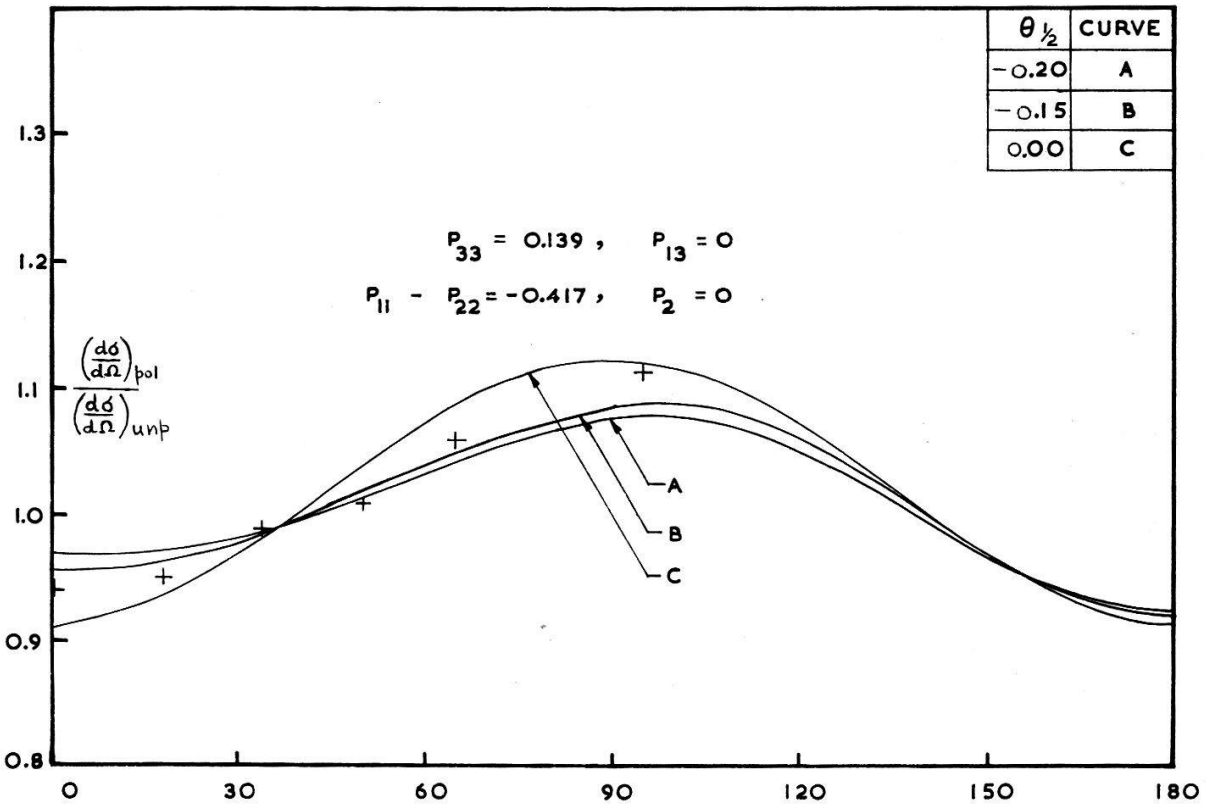

Figure 2

Polarized angular distributions for the $d+t$ reaction at $E_d = 500$ KeV for $\theta_{1/2} = -0.15$, -0.20, and 0, where $P_{33} = +0.139$ and $P_{11} - P_{22} = -0.417$.

Measurements corresponding to $\vartheta = \pi/2$ and $\varphi = \pi/2$ leads to $P_{33} = 0.139$, $P_{11} - P_{22} = 0.417$, $P_{13} = 0$, and $P_2 = 0.278$. Here, the inclusion of the stripping amplitude, as seen in figure 3, gives more agreement at the extreme angles but the resonance fit is, itself, reasonable at other angles. Also shown in this figure are the results for $\vartheta = \pi/2$ and $\varphi = 3\pi/2$ where the only change is a change in sign for P_2. The stripping contribution definitely leads to an improvement in this case. The effect of P_2 on the angular distributions is in fact very small as is revealed by the difference in intensities in the latter two measurements. Unfortunately the experimental errors are much more important and it is difficult to make any definite conclusion here. We recall that the s-wave resonance, by itself, results in $P_2^a = 0$. The effect of P_{13} is measurable by letting $\vartheta = \pi/2$ and $\varphi = 0$ and π, for example.

In summary, a simple characterization has been given of the $d + t$ reaction at deuteron energies near 500 KeV, which is consistent with the approaches both at higher and lower energies. The model is a crude one, owing to the uncertainties associated with the description of the deuterontriton interaction. The omission of spin-dependent distortion is, by itself, not felt to be too serious a shortcoming since a first-order treatment[17]) shows that the stripping contribution is insensitive to deuteron tensor polarization. There should be a contribution, however, owing to the interference with the resonance amplitude*).

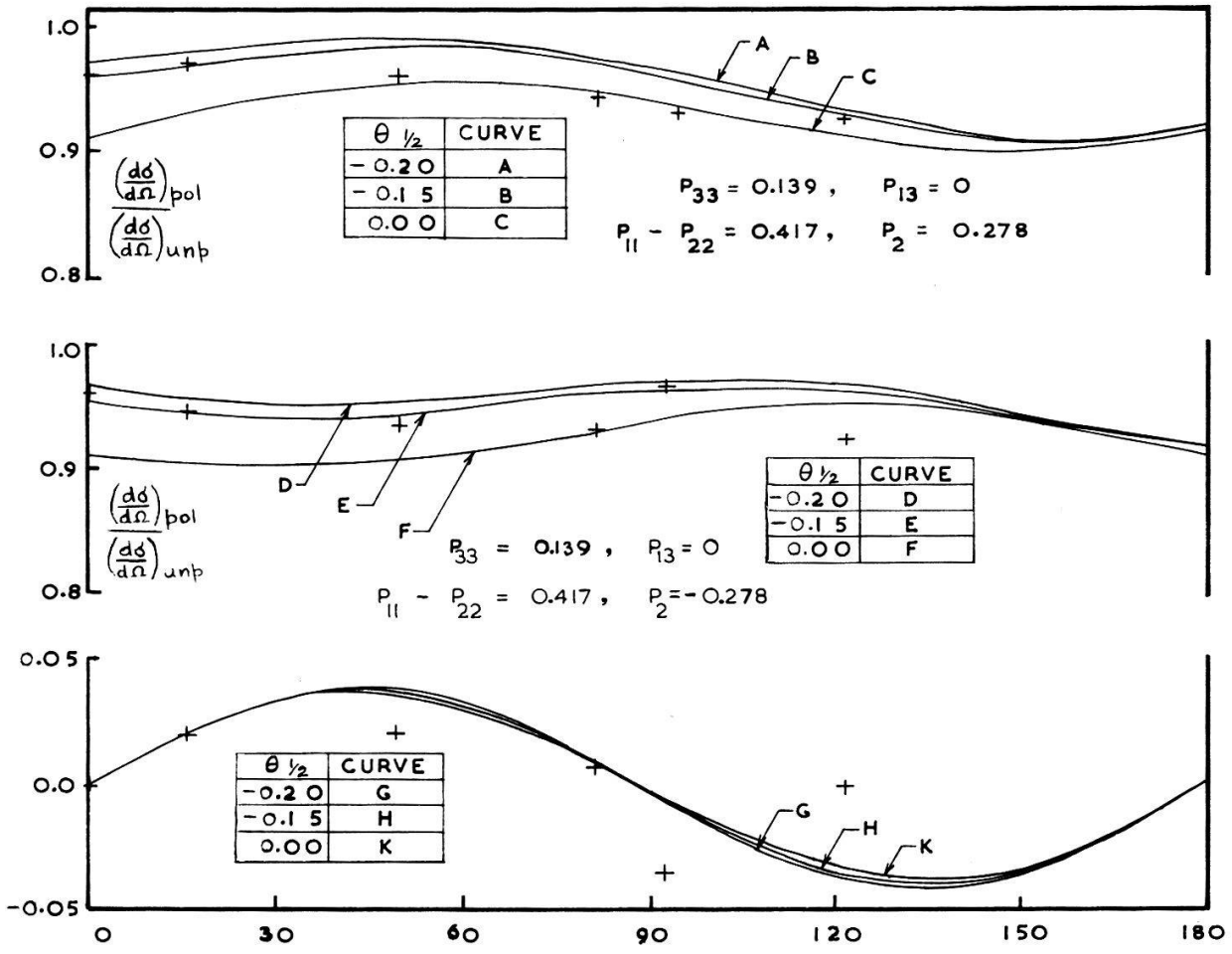

Figure 3

Polarized angular distributions for the $d+t$ reaction at $E_d = 500$ KeV for $\theta = -0.15, -0.20$, and 0, where $P_{33} = +0.139$, $P_{11} - P_{22} = 0.417$, and $P_2 = \pm 0.278$. Also shown is the difference of the two curves which is associated with the effect of P_2 only.

The calculations, based on this model, agree fairly well with the observed differential cross sections and the measurement of $P_{33}^a(\theta)$; although little improvement is provided by the added stripping amplitude to the fit with $P_{11}^a(\theta) - P_{22}^a(\theta)$. The effect of the deuteron vector-polarization is seen to be small. Generally, there is a range of angles where the efficiency tensors are little affected by the stripping contribution. The fact that the resonance contribution is still dominant in this energy range is of

*) The effect of the spin-dependent distortions demands a much more extensive numerical calculation, and this is currently being studied.

further interest. The s-wave resonance leads to efficiency tensors which are independent of E_d. Only a minor energy dependence would be expected with d-wave contributions owing to the Coulomb penetrability factors. If this energy dependence is measured and found to be small over an extended energy range, this would add greatly to the analyzing power of the reaction for deuteron tensor polarization, since we can then tolerate significant spreads in the deuteron energy.

On of us (A. H.) is indebted to the Colombo plan for the grant of a Fellowship.

Appendix

We wish to evaluate the matrix element $<(c\, s_2)\, S_2, l_2, b \mid R_D \mid (a\, s_1)\, S_1, l_1, b>$ for stripping processes with zero angular momentum transfer. If the distorted waves are normalized so that without distortion they are of the form $\exp(i\, k, r)$, then according to THOMAS[12])

$$R_D = -\frac{4\, i\, k_i\, k_f}{\hbar \sqrt{v_i\, v_f}} \frac{1}{(4\,\pi)^2} \quad V_D\,.$$

If we assume that there is no spin-dependent nuclear distortion, the distorted waves may be expanded in terms of partial waves with $l_1 = l_2$ and components $\lambda_1 = \lambda_2$ and the matrix element of V_D may be written as

$$< \mid V_D \mid > = \frac{1}{\hat{b}^2} \sum_{\beta} (a\, \alpha, s_1\, \sigma_1 \mid S_1\, \Sigma_1)\, (c\, \gamma, s_2\, \sigma_2 \mid S_2\, \Sigma_2)$$

$$(S_1\, \Sigma_1, l_1\, \lambda_1 \mid b\, \beta)\, (S_2\, \Sigma_2, l_2\, \lambda_2 \mid b\, \beta)\, (s_1\, \sigma_1 \mid s_2\, \sigma_2, s\, \sigma)\, (a\, \alpha, s\, \sigma \mid c\, \gamma)\, \theta_{1/2}\, A_{l_1}\, \delta_{l_1 l_2}\, \delta_{\lambda_1 \lambda_2}$$

where

$$A_l = (4\,\pi)^{3/2} \int_0^{\infty} r^2\, dr\, f_e^{(-)*}\,(k_f, r)\, u_p(r)\, f_e^{(+)}\,(k_i, r)\,.$$

The normalization, $(4\,\pi)^{3/2}$ in the radial integral is associated with factors $4\,\pi f_l(k, r)$ for each channel and a normalized function $(4\,\pi)^{-1/2}\, u_p(r)$ for the transferred radiation.

The summation over β and λ, leads to $S_1 = S_2$ and $\Sigma_1 = \Sigma_2$. Using the relation

$$\sum_{\Sigma_1} (a\, \alpha, s_1\, \sigma_1 \mid S_1\, \Sigma_1)\, (c\, \gamma, s_2\, \sigma_2 \mid S_1\, \Sigma_1) = (-)^{s_1 - c - \sigma_2}\, \frac{\hat{S}_1^{\,2}}{\hat{c}} \sum_{k\varkappa} \hat{k}\, (s_1\, \sigma_1, s_2 - \sigma_2 \mid k\, \varkappa)$$

$$(a\, \alpha, k\, \varkappa \mid c\, \gamma)\, W(a\, s_1\, c\, s_2;\ S_1\, k)\,,$$

and summing over σ_1, σ_2 and σ, we find

$$< \mid V_D \mid > = \delta_{s_1, s_2}\, \delta_{l_1 l_2}\, \theta_{1/2}\, A_{l_1}\, \hat{c}\, \hat{s}_1\, (-)^{\delta_1 - c - S_2}\, W(a\, s_1\, c\, s_2;\ S_1\, s)\,,$$

which is independent of b. For the $d + t$ reaction,

$$< \mid V_D \mid > = {}^1\!/_2\, \theta_{1/2}\, A_{l_1} \sqrt{3}\,.$$

If $l_p = l \neq 0$, the matrix element of V_D is written

$$< \mid V_D \mid > = \frac{1}{\hat{b}^2} \sum_{\beta \cdots} (a\, \alpha, s_1\, \sigma_1 \mid S_1\, \Sigma_1)\, (c\, \gamma, s_2\, \sigma_2 \mid S_2\, \Sigma_2)\, (s_1\, \Sigma_1, l_1\, \lambda_1 \mid b\, \beta)$$

$$(S_2 \Sigma_2, l_2 \lambda_2 \mid b\, \beta)\, (s_1\, \sigma_1, s_2\, \sigma_2 \mid s\, \sigma)\, (a\, \alpha, j\, m \mid c\, \gamma)\, (l\, \lambda, s\, \sigma \mid j\, m)$$

$$(l_1\, \lambda_1 \mid l_2\, \lambda_2, l\, \lambda)\, A_{l_1 l l_2}\, \theta_{jl}\,,$$

where

$$A_{l_1 l l_2} = (4\pi)^{3/2} \frac{\hat{l}_2 \hat{l}}{\hat{l}_1} (l_1 0 \mid l_2 0, l 0) \int_0^\infty r^2 \, dr \; f_{l_2}^{(-)*}(k_f, r) \, u_l(r) f_{l_1}^{(+)}(k_i, r) \, ,$$

This may be transformed to

$$\begin{aligned} \langle \mid V_D \mid \rangle = \sum_{jl} \frac{\theta_{jl}}{\hat{b}^2} A_{l_1 l l_2} \sum (-)^{b - S_1 - \lambda} \\ \frac{\hat{b}^2 \hat{k}}{\hat{S}_1} (l_2 \lambda_2, l_1 - \lambda_1 \mid k \varkappa) \, (s_2 \varepsilon_2, k \varkappa \mid s_1 \Sigma_1) \, W(s_2 l_2 S_1 l_1; b k) \\ (a \alpha, s_1 \sigma \mid S_1 \Sigma_1) \, (c \gamma, s_2 \sigma_2 \mid S_2 \Sigma_2) \, (s_1 \sigma_1 \mid s_2 \sigma_2, s \sigma) \\ (a \alpha, j m \mid c \gamma) \, (l \lambda, s \sigma \mid j m) \, (l_1 \lambda_1 \mid l_2 \lambda_2, l \lambda) \\ = \sum_{jl} \theta_{jl} A_{l_1 l l_2} \frac{\hat{l}_1 \hat{c} \hat{s}_1 \hat{j} \hat{S}_1 \hat{S}_2}{\hat{l}^2} (-)^{b - c + a - s - l_2 - S} \\ W(S_2 l_2 S_1 l_1; b l) \begin{Bmatrix} a & s_1 & S_1 \\ c & s_2 & S_2 \\ j & s & l_2 \end{Bmatrix} . \end{aligned}$$

This is seen to reduce to the formula given in the text for $\langle \mid V_D \mid \rangle$ when $l = 0$.

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