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Consistency of the Approach in Current Algebra [1])

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Abstract. The conserved vector current and the iso-triplet current hypotheses for the charged pion decay are used as a consistency check of the algebra of current, the partially-conserved axial-vector current and the soft-pion hypotheses and lead to the consistent result.

1. Introduction

Under the hypotheses of the algebra of currents [1] [2]) and the partially-conserved axial-vector current (PCAC) [2] ADLER [3] and WEISBERGER [4] have explained independently the axial-vector coupling-constant renormalization in the neutron beta decay. Since then there appeared many applications [5] [3]) in journals in a considerable success. All of those applications replace the physical amplitudes by the ones obtained by a suitable limiting procedure, so-called soft-pion limit [6]. The consistency conditions for this additional assumption was also discussed by ADLER [7] and recently by TOMOZAWA [8] in connection with the meson-baryon and meson-meson scattering processes.

In this paper we want to show that the assumptions of the algebra of current, the PCAC and the soft pion applied to the semi-leptonic $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ decay lead to the consistent result with the one obtained from the conserved vector current (CVC) and the iso-triplet current hypotheses [9] for this decay. The latter hypotheses are now well established experimentally [10] so that our proof is considered to be an additional consistency check to ADLER and TOMOZAWA's works.

In Section 2 the proof discussed above is shortly summarized.

2. Semi-Leptonic Decay of Charged Pion

We shall apply the algebra of current, the PCAC and the soft-pion hypotheses to the study of the isospin-changing hadronic current in the $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ decay

$$\pi^+ \rightarrow \pi^0 + e^+ + \nu_e \, . \tag{1}$$

The matrix element of the hadronic part of this decay can be described by the vector part of the hadronic currents in the normal $V - A$ theory due to the space reflection property of the pion. The small energy release in process (1) will not afford the induced

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[2]) Numbers in brackets refer to References, page 555.

[3]) We cite here only the papers related to the hadronic decay of hyperons.

interactions so that the primary vector current must be investigated. It transforms as the isospinchanging vector current density

$$J_\alpha^{V(-)}(x) = J_\alpha^{V1}(x) - i\, J_\alpha^{V2}(x) \tag{2}$$

in the language of the theory of algebra of current [1]. Here $J_\alpha^{Vi}(x)$ are the vector current density with the isospin component i. The matrix element of the space integral of the current (2) with respect to the initial and final pion states for process (1) is of interest. We shall work this matrix element out by the technique of dispersion theory.

Reducing the matrix element with respect to the neutral pion field, making use of the Klein-Gordon equation for free field, and performing the partial integration we arrive at the reduced formula

$$\left\langle \pi^0 \middle| \int d^3y\, J_\alpha^{V(-)}(y) \middle| \pi^+ \right\rangle = i\, \frac{1}{\sqrt{V}}\, \frac{1}{\sqrt{2\, k_0'}} \int d^4x\, e^{-i k' x}\, (\mu_\pi^2 - \Box_x^2)$$
$$\times \left\langle 0 \middle| T\left\{ \varphi^3(x) \int d^3y\, J_\alpha^{V(-)}(y) \middle| \pi^+ \right\rangle, \tag{3}$$

where V is the volume of quantization, k' is the four momenta of the pion, k_0' and μ_π are its energy and mass respectively, $\mu_\pi^2 - \Box^2$ is the Klein-Gordon operator, $T\{A\ B\}$ is the T-product, $\varphi^3(x)$ is the neutral-pion field, and $|\, 0 \rangle$ is the vacuum state.

The pion field $\varphi^i(x)$ and the axial-vector current density $J_\alpha^{Ai}(x)$ are related by the PCAC [2])

$$\varphi^i(x) = c\, \frac{\partial J_\mu^{Ai}(x)}{\partial x_\mu}, \tag{4}$$

where c is a constant independent of isospin. It is normalized as

$$c = \frac{g_r\, K^{NN\pi}(0)}{g_A\, m_N\, \mu_\pi^2}, \tag{5}$$

where g_r is the renormalized pion-nucleon coupling constant, $K^{NN\pi}(k^2)$ is its form factor normalized by $K^{NN\pi}(-\mu_{\pi)}^2) = 1$, g_A is the axial-vector coupling-constant renormalization, and m_N is the nucleon mass. As we shall see immediately that the explicit form of c is not necessary in our proof. Substituting (4) into (3) and taking the limit $k' \to 0$ after the partial integration, the right-hand side of Equation (3) reduces to the equal-time commutator of the axial-vector and the vector currents except for the kinematical factor. Hence we get

$$\lim_{k' \to 0} \sqrt{2\, k_0'} \left\langle \pi^0 \middle| \int d^3y\, J_\alpha^{V(-)}(y) \middle| \pi^+ \right\rangle = i\, \frac{1}{\sqrt{V}} \lim_{\mu_\pi \to 0} c\, \mu_\pi^2 \left\langle 0 \middle| \int d^3y\, J_\alpha^{A(-)}(y) \middle| \pi^+ \right\rangle, \tag{6}$$

where the use has been made

$$\left[\int d^3x\, J_0^{A3}(x), \int d^3y\, J^{V(-)}(y) \right]_{x_0 = y_0} = - \int d^3x\, J^{A(-)}(x)\,. \tag{7}$$

At this point we use the PCAC [2] and the result obtained from one-pion pole approximation

$$\left\langle 0 \middle| J_\alpha^{A(-)}(y) \middle| \pi^+ \right\rangle = - \frac{\sqrt{2}\, k_\alpha}{c\, \mu_\pi^2} \left\langle 0 \middle| \varphi^{(-)}(y) \middle| \pi^+ \right\rangle, \tag{8}$$

where k is the four momenta of the charged pion.

The right-hand side of Equation (9) may be reduced to the commutation relation of the creation and annihilation operators of the asymptotic fields with the appropriate coefficients since it is the matrix element of the renormalized pion field with respect to the one pion state and the vacuum. Substituting this into Equation (6) and again taking the limit $k \to 0$ we finally get

$$\lim_{\substack{k \to 0 \\ k' \to 0}} \left\langle \pi^0 \left| \int d^3y \; J^{V(-)}(y) \right| \pi^+ \right\rangle = + \frac{1}{\sqrt{2}} \, . \tag{9}$$

Under the CVC and the iso-triplet current hypotheses we have to normalize our current [9] by $\sqrt{2}\, G_V \, J^{V(-)}(y)$ in the starting formula. Thus the final result is exactly the consequence of the CVC and the iso-triplet current hypotheses although a different approach is used in the derivation. This may be considered as an additional consistency check [7, 8] for the physical assumptions made in many recent papers on the application of algebra of current.

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References

[1] M. GELL-MANN, Phys. Rev. *125*, 1067 (1962); Physics *1*, 63 (1964).
[2] M. GELL-MANN and M. LÉVY, Nuovo Cim. *16*, 705 (1960).
[3] S. L. ADLER, Phys. Rev. Lett. *14*, 1051 (1965); Phys. Rev. *140*, B736 (1965).
[4] W. I. WEISBERGER, Phys. Rev. Lett. *14*, 1047 (1965).
[5] M. SUZUKI, Phys. Rev. Lett. *15*, 986 (1965); H. SUGAWARA, *ibid.* *15*, 870 (1965); Y. HARA, Y. NAMBU, and J. SCHECHTER, *ibid.* *16*, 380 (1966); L. S. BROWN and C. M. SOMMERFIELD, *ibid.* *16*, 751 (1966); Y. T. CHIU and J. SCHECHTER, *ibid.* *16*, 1022 (1966).
[6] Y. NAMBU and D. LURIÉ, Phys. Rev. *125*, 1429 (1962).
[7] S. L. ADLER, Phys. Rev. *137*, B1022 (1965); *139*, B1638 (1965).
[8] Y. TOMOZAWA, *Axial-Vector Coupling-Constant Renormalization and Meson-Baryon Scattering Lengths*, Princeton preprint (April 1966).
[9] T. D. LEE and C. S. WU, Rev. nucl. Sci. *15*, 381 (1965); S. S. GERSHTEIN and J. B. ZELDOVICH, Zh. éksp. teor. Fiz. *29*, 698 (1955) (translation: Soviet Phys. JETP *2*, 576 (1957)); R. FEYNMAN and M. GELL-MANN, Phys. Rev. *109*, 193 (1958); R. E. MARSHAK and E. C. G. SUDARSHAN, *ibid.* *109*, 1860 (1958); J. J. SAKURAI, Nuovo Cim. *7*, 649 (1958); G. KÄLLÉN, *Elementary Particle Physics*, Addison-Wesley Pub. Com., INC. (1963).
[10] Ref. [184] in LEE and WU's article Ref. [9].